INSTRUCTION MANUAL

SD109B

CO/QUAD ANALYZER

MODIFIED TO MODEL SD109B-1

CPC 1756

SPECTRAL DYNAMICS CORPORATION

SERIAL 164

SDC F-20

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March 1970

(Revised April 1974)

3386

ADDENDUM

Model SD109B-1 Co/Quad Analyzer

The Model SD109B-1 is basically a standard SDC Model SD109B Co/Quad Analyzer. The "-1" added to the basic model number denotes a modification designed to provide enhanced vector product outputs. This is done by providing outputs AB cos θ and AB sin θ in the PRODUCT mode where the B input channel is artificially held constant over a change in input level of at least 20 dB. In the PRODUCT mode, this produces outputs KA cos θ and KA sin θ.

Operation in the Ratio Mode is unaffected by this modification. The outputs in the Ratio Mode are $\frac{A}{B}\cos\theta$ and $\frac{A}{B}\sin\theta$ in addition to absolute magnitude $\left|\frac{A}{B}\right|$.

Because of this modification, the SD109B-1 will not function for cross spectrum analysis. However, two individual channels of power spectrum analysis are available since operation in the square mode is not affected.

The internal changes resulting from this modification involved wiring of the S2B and S2C wafer sections of the front panel mode switch. The wires normally connected at the PROD position of S2B and S2C have been removed and jumper wires installed from the PROD position to the A/B ratio 100 position.

MANUAL CHANGES

Model: SD109B
Unit Serial: As Applicable
Manual Edition: --

Each change listed below must be individually analyzed for applicability in accordance with "Action" directed relative to unit serial number and/or documentation affected.

Change #1
ECO-1425
(3-9-73)

Effectivity: S/N 136 and on.

Action: Schematic, DWG. NO. 12271(E); On Sheet 2, Circuit Card A12 (Input Amplifier Assy. 12200-1), change R42 from 249Ω to 316Ω.

On Sheet 1, Circuit Card A6 (Co/Quad Channel A/B Assy. 12274-1),

		From	To
Change:	CR6	IN5231B	IN5235B
	V1	HP4523	VTL9A2
	R2	10k	30.1k
	R4	30k	3.01k
	R5	750k	49.9k
	R6	150k	49.9k
	R28	10k	100k
	R40 & R41	4.99k	49.9k

Add: Resistor R43 (10k, 1/8W, 1%)
Connected from the junction of R2/R5/C2 and ground.

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MODEL SD109B CO/QUAD ANALYZER

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APPENDICES

PS2: "Considerations in the Analysis of Arbitrary Waveforms"
By Anton C. Keller . . . . . Rear

Vector Component Techniques: A MODERN WAY TO MEASURE MODES
By Anton C. Keller . . . . . Rear

PREFACE

RECEIVING INSPECTION PROCEDURE

The Model SD109B Co/Quad Analyzer is shipped fully assembled, packed in a polystyrene case designed to protect it against all normal shipping hazards. Immediately upon receipt, inspect the exterior of the shipping case and note any visible damage. Keep all forms and invoices attached to the shipping case. Remove the instrument from the shipping case and test its functional operation in accordance with Section II of this manual. If shipping damage is apparent, file a claim with the carrier's claims agent and send a copy to Spectral Dynamics Corporation. Be sure to include the instrument name, model number, and serial number on all correspondence. Spectral Dynamics Corporation will advise the buyer what should be done; arrangements for repair or replacement will be made as applicable.

WARRANTY POLICY

All Spectral Dynamics products are warranted against defects in materials and workmanship. This warranty applies for one year from the date of delivery, or, in the case of certain components listed in some of the operating manuals, for the specified period. We will repair or replace products which prove to be defective during the warranty period, providing the failure or damage has not been caused by misuse or abnormal operating conditions. Fuses and lamps are specifically excluded from warranty. If the malfunction or a portion thereof is determined by Spectral Dynamics Corporation to have been caused by misuse or abnormal conditions of operation, an estimate of cost to repair will be submitted to the purchaser for approval before beginning any repair work.

Liability under this warranty is limited to servicing and adjusting the instrument returned to the factory, with transportation charges prepaid by the purchaser. No other warranty is expressed or implied. We are not liable for consequential damages.

If any malfunction develops, notify Spectral Dynamics Corporation or its representative in your area, giving details of the problem, and the name, model and serial number of the equipment. Upon receipt of this information, service data or shipping instructions will be provided.

RETURN SHIPMENT PROCEDURE

On receipt of shipping instructions, forward the instrument prepaid via air freight to the factory or authorized repair station indicated on the instructions. Replace the instrument in its original shipping case, or surround with two to three inches of shock absorbing material and then pack it in a heavy-weight cardboard box, wooden box, or other strong container.

SECTION I

GENERAL INFORMATION

1.1 INTRODUCTION

The Model SD109B Co/Quad Analyzer automatically calculates the real and imaginary product, or ratio relationships between two 100-kHz filtered analog input signals. These signals, normally supplied from SDC Dynamic Analyzer and/or Tracking Filter units, are amplitude and phase coherent with original audio data signals. (See Figure 1-1.)

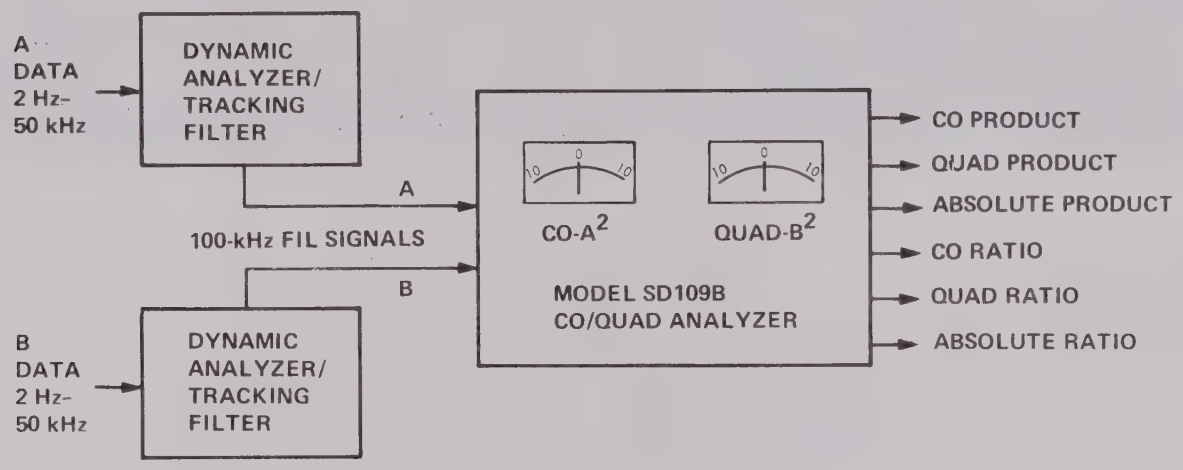


Figure 1-1. Model SD109B Co/Quad Analyzer System Concept

The Model SD109B employs analog multiplication, division, and averaging to provide front-panel meter displays of both the coincident and quadrature products and ratio. These products along with the absolute products and ratio are available as proportional d-c output signals suitable for driving external recording and monitoring equipment.

The 100-kHz filtered signal outputs from the SDC Dynamic Analyzer Models SD101A, SD101AS, SD101B, and SD101BS, and Tracking Filter Models SD1012 and SD1012B, are specifically compatible with the requirements of the Model SD109B.

1.2 FRONT PANEL FUNCTIONAL DESCRIPTION

Figure 1-2 illustrates the instrument front panel followed by a brief description of each of the front panel functions.

Figure 1-2. Front Panel

POWER

Push button type switch/indicator used to turn instrument on and off.
Push button glows red when power is on.

CO-A^2 Meter

Displays coincident products of A and B inputs as selected by the mode switch. In CAL CO mode, displays A^2; in CAL QUAD, B x B cos 90°; in OPER SQ, A^2; in OPER PROD, A x B cosθ; and in OPER RATIO, (A/B) cosθ. (θ is the phase angle between A and B inputs.)

FULL SCALE CAL (CO)

Used to calibrate CO-A^2 meter to full scale with mode switch set to CAL CO position; provides approximately 5 dB total attenuation of input A signal.

CO-A^2 AVG (SEC)

Ten-turn multi-dial control and multiplier switch (X0.1 and X1.0) used for time averaging the d-c output and CO-A^2 meter response; overall range is 0.1 to 10.0 seconds. Time can be extended by connecting an external resistor across rear-chassis connectors J11 and J12. (Refer to paragraph 2.2.4, Section II.) Averaging affects the following modes: CAL CO (A^2), OPER PROD (A x B $\cos\theta$), OPER SQ (A^2), and OPER RATIO [(A/B) $\cos\theta$].

A INPUT Range Switch

Used to select full-scale range of A input signal; calibrated in 5 dB steps over the range of -30 dB (31.6 mV rms) to +10 dB (3.16 Vrms); 0 dB is referenced to 1 Vrms full scale input.

A OVERLOAD

Lights if signal A input exceeds full scale by 10 percent (approximately 1 dB).

QUAD-B^2 Meter

Displays the quadrature products of A and B inputs as selected by the mode switch. In CAL QUAD mode, displays B^2; in CAL CO, B x B cos 90°; in OPER PROD, A x B $\sin\theta$; in OPER SQ, B^2; and in OPER RATIO, (A/B) $\sin\theta$.

FULL SCALE CAL (QUAD)

Used to calibrate QUAD-B^2 meter to full scale with mode switch set to CAL QUAD position; provides approximately 5 dB total attenuation of input B signal.

QUAD-B^2 AVG (SEC)

Control and multiplier switch (X0.1 and X1.0) used for time averaging the d-c output and QUAD-B^2 meter response; overall range is 0.1 to 10.0 seconds. Time can be extended by connecting external resistor across rear-chassis connectors J2 and J3. (Refer to paragraph 2.2.4, Section II.) Averaging affects the following modes: CAL QUAD (B^2), OPER PROD (A x B sineθ), OPER SQ (B^2), and OPER RATIO [(A/B) sinθ)].

B INPUT Range Switch

Used to select full-scale range of B input signal; calibrated in 5 dB steps over the range of -30 dB (31.6 mV rms) to +10 dB (3.16 Vrms); 0 dB is referenced to 1 Vrms full-scale input.

B OVERLOAD

Lights if signal B input exceeds full scale by 10 percent (approximately 1 dB).

ABS AVG (SEC)

Control and multiplier switch (X0.1 and X1.0) used for time averaging the d-c output and meter response of the selected channel; overall range is 0.1 to 10.0 seconds. Time can be extended by connecting an external resistor across rear-chassis connectors J6 and J7. Averaging affects the absolute value outputs of OPER PROD (|A x B|), OPER SQ (A^2), and OPER RATIO (|A/B|).

Mode Switch

CAL CO	With full-scale A and B inputs applied, CO-A^2 meter is calibrated for full scale using FULL SCALE CAL (CO) adjust. QUAD-B^2 meter reads zero (B x B cos 90°), and 10 Vdc full-scale output is present at CO output J13.
CAL ZERO	Both front-panel meters read zero and all outputs are 0 Vdc.
CAL QUAD	With full-scale A and B inputs applied, QUAD-B^2 meter is calibrated for full scale using FULL SCALE CAL (QUAD) adjust. CO-A^2 meter reads zero (B x B cos 90°) and 10 Vdc full-scale output is present at QUAD output J4.
OPER PROD	CO-A^2 meter reads A x B cosθ; QUAD-B^2 meter reads A x B sinθ. All rear chassis d-c outputs are available.
OPER SQ	CO-A^2 meter reads A^2; QUAD-B^2 meter reads B^2. All rear-chassis d-c outputs are available; the absolute output will provide $\lvert A \times A \rvert$.
OPER FULL SCALE RATIO (A/B)	The following meter readings and d-c output are provided: CO-A^2: (A/B) cosθ QUAD-B^2: (A/B) sinθ ABS: $\lvert A/B \rvert$ In any position (1,10, or 100), full-scale output is 100 mV indicating that the A/B ratio is 1,10, or 100 as selected.

1.3 REAR CHASSIS FUNCTIONAL DESCRIPTION

Figure 1-3 illustrates the rear of the instrument chassis followed by a brief description of each of the input/output functions.

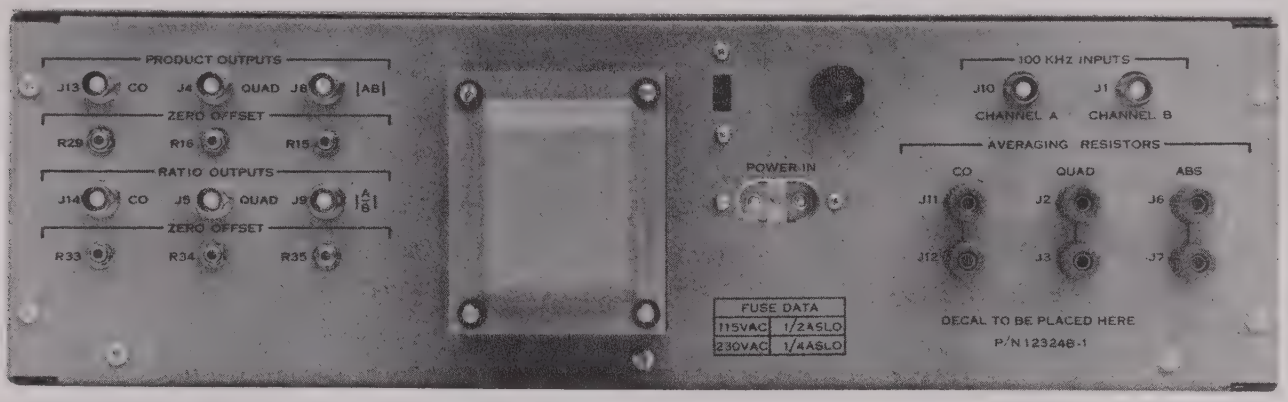


Figure 1-3. Rear Chassis

OUTPUTS

Table 1-1 gives the outputs available at each connector for each setting of the front-panel Mode switch. These outputs assume the application of 100-kHz signal inputs from a Model SD101A, SD101B, SD1012B or equivalent Dynamic Analyzer or Tracking Filter:

Input A = A cos (ωt +θ), and Input B = B cos (ωt).

Table 1-1. Outputs

Output Connector		Output for Mode Switch Setting					
		CAL			OPER		
		CO	ZERO	QUAD	PROD	SQ	F.S.RATIO A/B (1,10, or 100)
CO	(J13)	A^2	0	*0	AB cos θ	A^2	---
QUAD	(J14)	*0	0	B^2	AB sin θ	B^2	---
|A x B|	(J8)	A^2	0	B^2	|A x B|	A^2	---
RATIO CO	(J14)	$A^2/100$	0	*0	---	---	(A/B) cos θ
RATIO QUAD	(J5)	*0	0	$B^2/100$	---	---	(A/B) sin θ
RATIO |A/B|	(J9)	$A^2/100$	0	$B^2/100$	---	---	|A/B|
*0 = (B cosωt) x (B cos (ωt + 90°))							

PRODUCT ZERO OFFSET

Used during CAL ZERO mode to set zero output reference level of associated output connector (located directly above each control).

RATIO ZERO OFFSET

Used during RATIO mode to set zero output reference level of associated output connector (located directly above each control).
To set zero level, A input must be shorted and B input at full scale.

CHANNEL A (J10), CHANNEL B (J1), 100 kHz INPUTS

Each input accepts the 100-kHz filtered signal from an SD101A, SD101B, SD1012B, etc., SDC Dynamic Analyzer or Tracking Filter. These 100-kHz signals are amplitude and phase coherent with the audio frequency data signal being analyzed.

CO (J11,-12), QUAD (J2,-3), ABS (J6,-7) AVERAGING RESISTORS

Used for connecting external resistors to extend time averaging of designated output. ((Refer to front-panel description of CO-A^2 AVG (SEC) control, etc.)) If used, the internal jumper across the particular pair of terminals must be unsoldered. Refer to paragraph 2.2.4, Section II.

FUSE

Contains 3AG type SLO-BLO fuse; ½A for 115V operation and ¼A for 230V operation.

POWER IN

Accepts 3-wire plug and cable (supplied) for applying 115/230 Vac 50/60 Hz primary input power. Center terminal is chassis ground. Uses Belden No. 17258 cable.

115/230 Switch

Facilitates changeover of primary power input circuit for either 115V or 230V operation. Proper input voltage to be applied shows in window; screwdriver blade slot is up for 115V operation and down for 230V operation.

1.4 SPECIFICATIONS

SYSTEM ANALYSIS RANGE . . . . . 2 Hz to 50 kHz (Using SDC Dynamic Analyzer or Tracking Filter units)

INPUTS (Each of two channels)

Frequency . . . . . 100 kHz (fixed)

Amplitude . . . . . 31.6 mV to 3.16 Vrms for full-scale output. 40 dB input attenuator calibrated in 5 dB steps.

Impedance . . . . . 10 kilohms

Dynamic Range (Product |AxB|) . . 60 dB

(Ratio |A/B|) . . . . 50 dB
Ratios of 0.1 to 100 in three ranges
Ratio 1 . . . . . B minimum ≧ 0.1B full scale
Ratio 10,100 . . . B minimum ≧ 0.01B full scale

OUTPUTS

Product Mode . . . . . –10V to +10 Vdc for both CO and QUAD function
0 to +10 Vdc for absolute product

Accuracy . . . . . ±0.25% of full scale ±2.5% of reading

Square Mode . . . . . 0 to +10 Vdc for both CO and QUAD function

Accuracy . . . . . ±0.1% of full scale ±2.5% of reading

Dynamic Range . . . . . 80 dB

Ratio Mode . . . . . –100 mV to +100 mV dc for both CO and QUAD function
0 to 100 mV for absolute ratio

Accuracy . . . . . ±2% of full scale ±5% of reading

Impedance . . . . . Less than 600 ohms

MISCELLANEOUS

METERS	Front-panel meters continuously monitor CO and QUAD components. Meters are graduated in 100 divisions between limits of +10 and -10.
Linearity	2% of full scale
TIME AVERAGING	Continuously adjustable 0.1 to 10 seconds, ±15% for each channel (For extension of averaging time refer to paragraph 2.2.4.
INPUTS/OUTPUTS	BNC Connectors
POWER	105-125/210-250 Vac, 50-60 Hz, at ≈60W; input line voltage switch-selectable at rear of chassis

DIMENSIONS

Height	5¼" (13,3 cm)
Width	19" (48,3 cm)
Depth	14" (35,6 cm)
Shipping Weight	≈20 lbs (9 kg)

SECTION II

OPERATION

2.1 GENERAL

The primary operating procedure described here is basic to the SD109B Co/Quad Analyzer. Interpretation of the data will vary, and additional system calibration functions may be required depending upon the specific application. If the instrument is furnished as part of an SDC system, refer to the particular system instruction manual for installation, calibration and operating instructions applicable to system operation. Refer to Section I of this manual for the description of all front panel and rear chassis connectors, controls, switches and indicators. The following operating procedure is for use with Models SD101B, SD101BS and SD1012B Dynamic Analyzer/Tracking Filter (DA/TF) units.

2.2 OPERATION

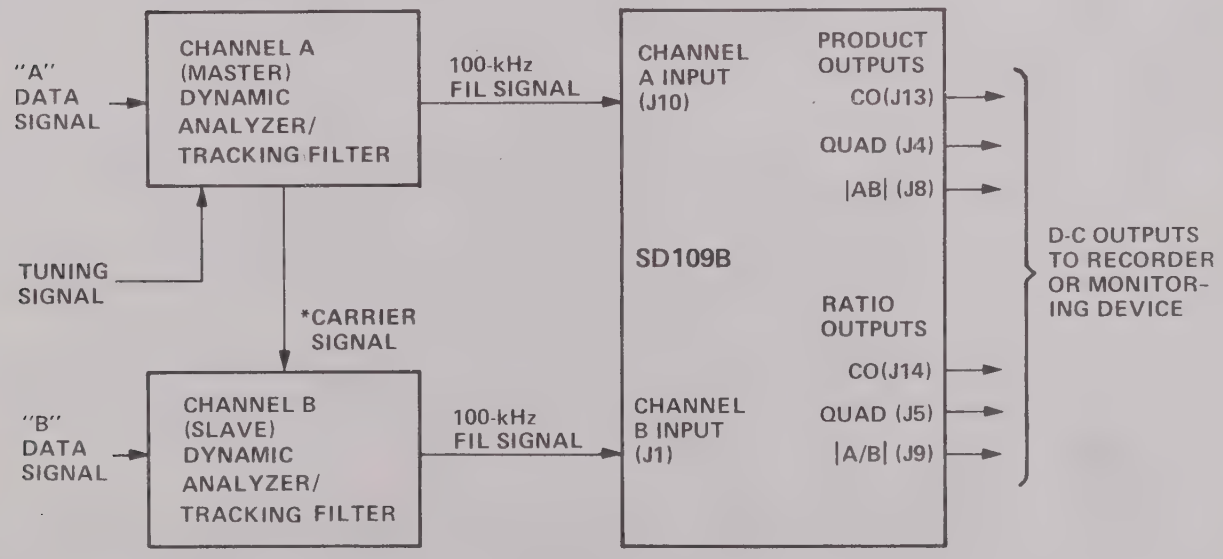
2.2.1 Preliminary Powering

a. Check that the 115/230 switch on the instrument rear chassis is set to indicate the proper input line voltage. Check that the proper fuse is installed.

b. Press the front panel power switch/indicator; it should light indicating power is on. Allow at least ½ hour warm-up.

2.2.2 Operation with SD101B/SD101BS and/or SD1012B Dynamic Analyzer/Tracking Filter Unit

a. Interconnect the SD109B and DA/TF units as shown in Figure 2-1.



*For SD101B/SD101B or SD101B/SD101BS combination, connect CARRIER OUT of Channel A unit to CARRIER IN of Channel B unit. For SD1012B, an SD1010B Carrier Generator provides a common carrier signal to both Channels A and B.

Figure 2-1. CO/QUAD Analysis System

b. Calibrate the tracking filters in accordance with the normal procedures for the instruments being used. You may wish to refer to the individual instruction manuals for the particular DA/TF units to insure that correct 100 kHz filtered signal inputs are being furnished to the SD109B CHANNEL A and CHANNEL B inputs.

c. Set the DA/TF units in internal CAL mode. The filtered 100-kHz output level will be approximately 1.8 Vrms, and represents the full scale data level. (For SD101A, SD101AS, SD1012, the filtered 100-kHz output level will be approximately 700 mV rms, and represents 0.707 times the full scale data level.

d. Set the SD109B A INPUT and B INPUT switches to the +5 dB range position. This should be the correct range setting for a full scale filtered 100-kHz level for most DA/TF units of the "B" family. (When using either SD101A, SD101AS, or SD1012 units, the zero dB settings of the SD109B input range attenuators would normally be used.) The general rule is to note that 0 dB is referenced to 1 Vrms and to set the SD109B input range switches to the setting closest to the actual full scale input; −5 dB for 0.56 Vrms, −10 dB for 0.32 Vrms and so forth. This is necessary if the maximum dynamic range of analysis is to be achieved.

e. Set the SD109B Mode switch to CAL ZERO position. If necessary, adjust the ZERO OFFSET controls on the rear panel of the SD109B for a zero indication on both meters. For greater accuracy, monitor the CO and QUAD PRODUCT OUTPUTS with a DVM while adjusting R29 and R16.

f. Set the SD109B Mode switch to the CAL CO position. If necessary, adjust the CO FULL SCALE CAL control (under the CO-A^2 meter) for an indication on the meter of +10.0, the normalized setting for 1.0 x 1.0. (Using Models SD101A, meter indication should be +5, the normalized setting for 0.707 x 0.707.) The QUAD-B^2 meter should read zero.

g. Set the SD109B Mode switch to the CAL QUAD position. If necessary, adjust the QUAD FULL SCALE CAL control (under the QUAD-B^2 meter) for an indication on the meter of +10.0.

h. Set the SD109B Mode switch to the OPERATE PROD position. Adjust the phase controls of the tracking filters for zero on the QUAD-B^2 meter. The CO-A^2 meter should now read +10.0. If it reads −10.0, the phase has been normalized to 180° rather than zero

degrees and the 0–180 phase switch on either the Channel 1 or Channel 2 tracking filter should be thrown and steps f. and g. repeated as necessary.

i. One additional check should be made to insure that an absolute zero phase does, in fact, exist between the tracking filter channels. Using the internal cal mode, there is no certainty that in the individual SD101/SD1012 channels the 60-Hz calibration transformer windings are tapped identically, as to phase. A positive check can be made by returning the DA/TF units to the OPERATE mode and applying a common sinusoidal input (at the same frequency as the tuning signal) to both DA/TF signal inputs. Common signal inputs should result in a positive CO indication and zero QUAD reading. The phase adjust control on either DA/TF unit may be adjusted again to obtain the proper meter readings. Readjust CO and QUAD FULL SCALE CAL if necessary.

j. To set up for RATIO mode, set Mode switch to RATIO 10, temporarily short CHANNEL A INPUT (J10), apply full-scale input signal to CHANNEL B INPUT (J1), and adjust RATIO ZERO OFFSET controls for zero level output at RATIO OUTPUTS (J14, J5 and J9). Remove temporary short at J10 and restore CHANNEL A input.

NOTE

The SD109B is temperature stable to within 0.05% of full scale output per degree centigrade; however, for wide variations in ambient temperature, repeat the steps outlined in the preceding paragraphs 2.2.2 e. and 2.2.2 j. as necessary.

The SD109B and DA/TF units are now calibrated to operate as a CO/QUAD analysis system. Adjust the DA/TF units for operate mode as described in their respective manuals.

NOTE

In the remainder of this section the use of SD101B, SD101BS or SD1012B, DA/TF units is assumed. If SD101A, SD101AS or SD1012 units are used, alter the procedures accordingly, as demonstrated in the preceding paragraphs of this section.

The following EXAMPLE provides an interpretation of meter readings and outputs from the SD109B for typical input signal levels derived from two accelerometers.

EXAMPLE: Assume that the signal outputs from two accelerometers with sensitivities of 10 mV_{rms}/g_{rms} are applied to the DA/TF units. The maximum g level obtained during the test is 10 g_{rms}. The preceding calibration procedure establishes the range setting of the DA/TF at 100 mV_{rms} (10 mV/g x 10g = 100 mV), which is 10 g_{rms} full scale.

With the DA/TF units in CAL mode, the 100-kHz filtered signal outputs represent full scale, or 10 g_{rms}. Thus the CO-A^2 meter and CO PRODUCT OUTPUTS represent 10g x 10g = 100 g^2 or CO full scale.

2.2.3 Operation with a Recorder

As described in Section I (paragraph 1.2.3 OUTPUTS, and Figure 1-3, Rear Chassis Description), d-c voltages proportional to the CO and QUAD products and ratios are provided for driving an X-Y recorder.

A frequency response plot of any output function can be made as follows: Apply the selected function output to the recorder Y-axis. To the X-axis, apply a dc proportional to frequency, normally available from the sweep oscillator that is being used to tune the DA/TF units. For example, the SDC Model SD104 Sweep Oscillator and Model SD114 Sweep Oscillator Servo provide both a log and linear dc proportional to frequency output.

Calibrate the recorder plot for zero and full scale using the procedure outlined in paragraph 2.2.2.

To make a polar plot, apply the CO PRODUCT OUTPUT to the recorder X-axis and the QUAD PRODUCT OUTPUT to the Y-axis.

If desired, the d-c polarity of the CO or QUAD PRODUCT OUTPUT can be reversed as follows: The SDC family of DA/TF units are each equipped with a 0-180 phase reversal switch (located inside the chassis). To reverse the CO PRODUCT OUTPUT polarity, first normalize the A and B CHANNEL inputs to the SD109B for zero degree phase difference. Then change the 0-180 switch position, and the CO PRODUCT OUTPUT will change from positive to negative.

To reverse the QUAD PRODUCT OUTPUT polarity, first adjust the A and B CHANNEL inputs for 90 degrees phase difference. Then change the 0-180 switch position, and the QUAD PRODUCT OUTPUT will change from positive to negative.

2.2.4 Extending Averaging Time

Normally, 10 seconds is the maximum averaging time selectable on the front panel, using the AVG(SEC) control for the CO, QUAD and ABS functions. Averaging time may be extended beyond the maximum selected on the front panel by connecting resistance (50 kΩ for each additional second) across the appropriate terminals on the rear of the chassis.

NOTE

An internal factory-installed jumper across each set of terminals, just inside the rear of the chassis wall, must also be removed.

2.3 APPLICATIONS

2.3.1 Cross Spectrum Analysis

The cross spectrum ($P_{12}[\omega]$) relating two random signals is a complex valued function, having both a modulus (magnitude) and argument (phase) term that may be expressed as follows:

$$P_{12}(\omega) = |P_{12}(\omega)| \angle \theta(\omega)$$

or,

$$P_{12}(\omega) = C_{12}(\omega) + j\,Q_{12}(\omega)$$

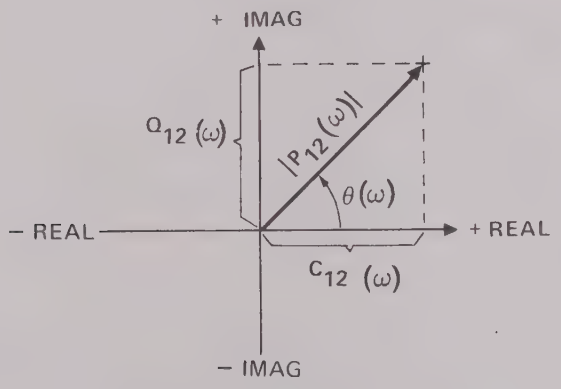
$C_{12}(\omega)$ is the coincident or in-phase portion of the complex product and $Q_{12}(\omega)$ is the quadrature or 90° portion of the same product. These terms are related by:

$$P_{12}(\omega) = [C_{12}(\omega)^2 + Q_{12}(\omega)^2]^{\frac{1}{2}}$$

or

$$\theta(\omega) = \tan^{-1} \frac{Q_{12}(\omega)}{C_{12}(\omega)}$$

This may be illustrated as follows:



It may be assumed that the signals under analysis are acceleration waveforms represented by $a_1(t)$ and $a_2(t)$. After these signals have been processed through the DA/TF units, the filtered signal outputs (both audio and 100-kHz IF outputs) represent the Fourier transform of the acceleration inputs. The analyzer outputs may be expressed as follows.

$$A_1(\omega) \text{ and } A_2(\omega)$$

To form the cross spectrum between $a_1(t)$ and $a_2(t)$ is essentially analogous to forming the product $A_1(\omega)\,A_2^*(\omega)$ where $A_2^*(\omega)$ is the complex conjugate of $A_2(\omega)$.

EXAMPLE: Two SD101B Dynamic Analyzers are connected to the SD109B. Set up as follows:

10 mV rms/g rms input sensitivity
10g rms total g level
5 Hz filter bandwidth in each SD101B
10 Hz to 5 kHz frequency range

100 mV rms (SD101B) full scale range will be used during analysis.

METER MULTIPLIER X 1 (SD101B)

$$\text{Full Scale (SD101B)} = \frac{\text{RANGE}}{\text{SENSITIVITY}}$$

$$\text{F.S.} = \frac{100}{10} = 10\text{g rms}$$

Using internal CAL on each analyzer and SLAVED CARRIER tuning, set each SD101B for full scale. (See SD101B instruction manual for additional details.)

Calibrate the system by performing steps a. through h. of paragraph 2.2.2.

The SD109B outputs now represent 20g rms^2/Hz.

$$\text{CAL} = \frac{g_1 \text{ rms} \times g_2 \text{ rms}}{\text{BW eff.}}$$

$$= \frac{10\text{g rms} \times 10\text{g rms}}{5 \text{ Hz}}$$

$$= 20\text{g rms}^2\text{/Hz}$$

Set up the DA/TF units for normal operating mode. It is assumed that random noise from either an on-line test or a magnetic tape is applied to their inputs. On the DA/TF units, select an input range that affords maximum sensitivity without overload. (A normal range setting may cause occasional OVERLOAD flashing.) If both A and B input range switches are in the +5 dB position, the calibration point previously set on the plotter remains 20g rms^2/Hz.

If both A and B input range switches are moved to −5 dB, the calibration point becomes 2g rms^2/Hz; if each has been moved to −15 dB, the CAL point becomes 0.2g rms^2/Hz. Note that the CO/QUAD outputs are bipolar and are, therefore, normally plotted in linear fashion.

2.3.2 Summary

For PSD measurements using SD101B and log output:

$$\text{Full Scale} = \frac{\text{RANGE}}{\text{SENSITIVITY}}$$

$$\text{CAL} = \frac{(\text{F.S.} \times 1.13)^2}{\text{BW eff.}}$$

The constant "1.13" takes into account the difference between absolute average and rms values of a Gaussian waveform using an average detector. See SDC technical publication (PS-2), "Considerations in the Analysis of Arbitrary Waveforms" for additional details.

For cross spectrum measurements using two SD101B units and an SD109B:

$$\text{(SD101B) Full Scale} = \frac{\text{RANGE}}{\text{SENSITIVITY}}$$

$$\text{(SD109B) CAL} = \frac{\text{F.S.}_1 \times \text{F.S.}_2}{\sqrt{\text{BW}_1 \text{ eff} \times \text{BW}_2 \text{ eff}}}$$

Since the SD109B uses a direct multiplication technique followed by a low-pass filter, the resulting d-c component is directly proportional to the mean square value of the signal input product. No detector as such is required for cross measurements.

2.3.3 Interpretation of Cross Spectrum Measurements

One of the most useful applications for cross spectrum measurements is the establishment of system transfer functions. The complex transfer function ($H[j\omega]$) of a system is related to the cross spectrum ($P_{12}[\omega]$) by:

$$P_{12}(\omega) = P_{11}(\omega)H(j\omega)$$

where

$P_{11}(\omega)$ is the input power spectrum.

If a signal input of white noise is used such that the input spectrum is "flat", it can be seen that the transfer function and cross spectrum are related by a constant.

This is shown pictorially in Figures 6 and 13 of the enclosed SDC technical publication (PS-3), "Some Uses of Real and Imaginary Component Vectors in Engineering Analysis". Figure 6 indicates the transfer function for a linear electronic system, in terms of CO and QUAD components, as excited by a swept-sine function. Figure 13 shows the cross spectrum between input and output of the same system, when white noise excitation is used. The two figures are related and very nearly identical.

2.3.4 Modal Analysis

One of the primary applications for the Model SD109B Co/Quad Analyzer is in the analysis of structural mode shapes. This is especially true when force and acceleration measurements are used. There are two fundamental reasons for the use of real and imaginary vectors (Co and Quad components) for determining structural mode shapes. First, the choice of the appropriate vector component gives a direct indication of the relative amplitude and direction of a mode shape at a selected resonant frequency. Secondly, the vector technique has shown a significant enhancement in terms of decoupling adjacent modes, such as bending and torsional modes or adjacent bending modes.

The idea of using the vector technique in modal separation evolves from the theory of exciting pure modes. Any structure responds as the sum of its normal modes. If a pure mode is excited in a structure, the structural deformation response takes on a unique pattern. This is shown in Figure 2-2.

If a structure is excited into a pure mode, its response curve approximates that of a single degree of freedom. Figure 2-2(a) shows the ratio of response acceleration to input force for a system with one resonant frequency. The corresponding phase response for this system is seen in Figure 2-2(b). Here the phase angle begins at 0 degrees when force and acceleration are in phase below resonance, goes through 90 degrees at resonance and achieves a final phase relationship of 180 degrees above resonance. If the response in terms of acceleration to force ratio is broken down into real and imaginary components, rather than to overall magnitude and phase, the response produces vector components as seen in Figures 2-2(c) and 2-2(d). The real part of the response vector is seen to peak up before and after resonance and go through zero at resonance. At the same time, the quadrature or "resonant" vector peaks up sharply at resonance and decays to zero at either side of resonance.

If a pure mode is excited in a structure and the excitation frequency is swept from below resonance through the resonant condition to a frequency above resonance, a Co vs Quad (Nyquist) diagram, as seen in Figure 2-2(e), will result. For a linear system with viscous damping, a plot of the coincident response vector vs the quadrature response vector will produce a perfect circle.

Examining these vector response curves in detail verifies one of the classic definitions of a pure mode. If a pure mode of vibration is excited, every point on the "modal surface" will be exactly in phase or exactly out of phase with every other point on this surface. If a response vector of acceleration is compared with an input force vector, then every point on the modal surface will be either (+) or (–) 90 degrees with the reference force using these

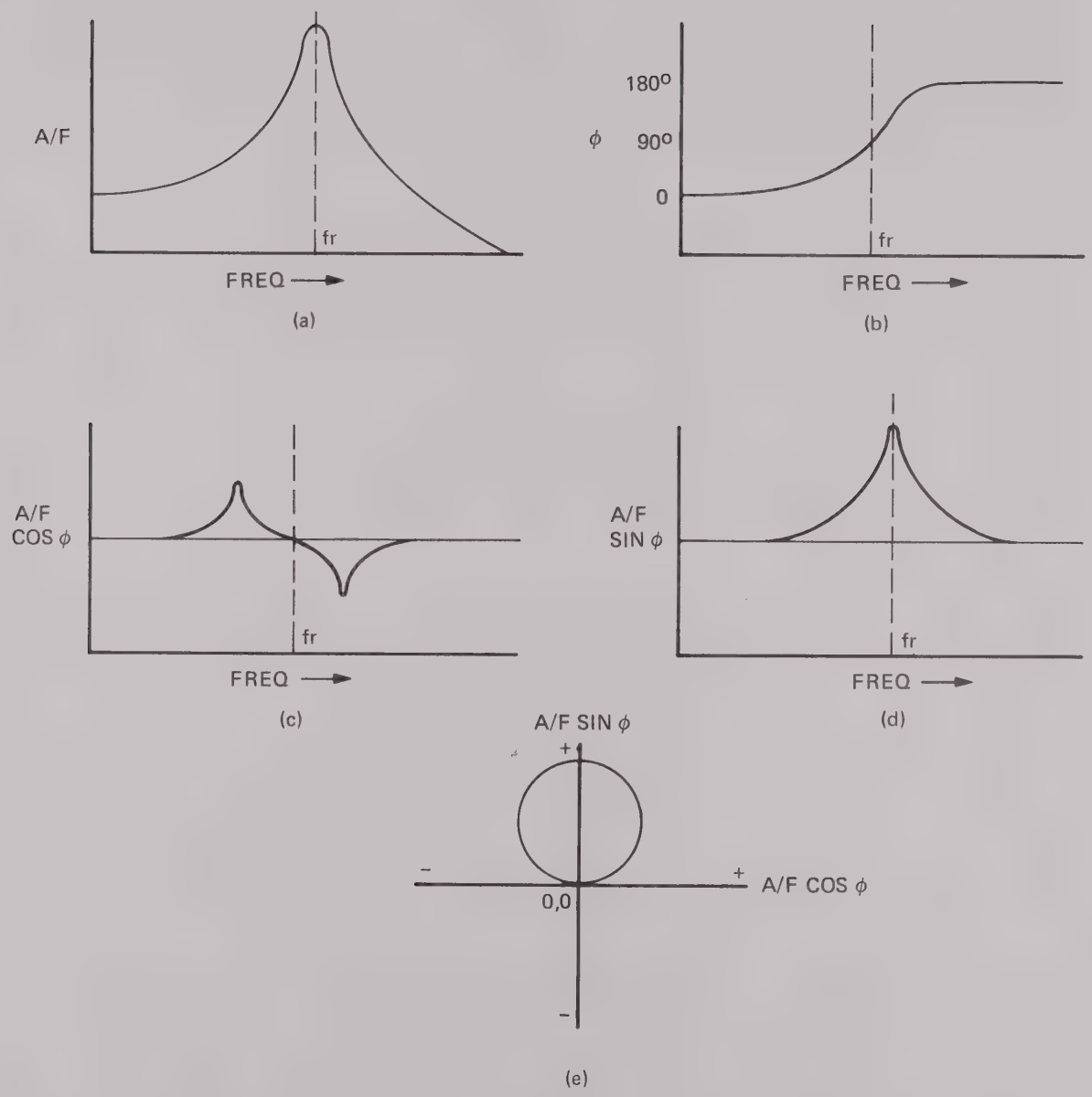


Figure 2-2. The Vector Technique — Force and Acceleration Measurements

measurement terms. Even if a pure mode is not excited, if force and acceleration terms are used, the magnitude of the quadrature vector will indicate only that amount of response in the direction of the mode being isolated. In other words, mode shapes can be accurately determined using the Co and Quad vector technique even if a pure mode is not excited. Mode shape measurements then reduce to the measurement of the term A/F sin θ. If the input force to a structure is controlled to a constant level, the measurement can be simplified even further to determine that vector A sin θ. If the force input to a structure is controlled to a constant level, the response measurements FA sin θ, or A/F sin θ, will produce identical results.

In actual use, the Model SD109B Co/Quad Analyzer accepts filtered 100 kHz I.F. input signals from two channels of Spectral Dynamics tracking filters. These can take the forms of many different varieties of tracking filters as seen in Figure 2-3. As the two filtered signals are applied to the Co/Quad Analyzer, one of them is phase shifted 90 degrees. Two products between the input signals are then formed, one with the phase-shifted terms included. Simultaneous outputs proportional to the in phase (Co) and quadrature (Quad) components are then produced in addition to meter readouts of these same quantities. Note that, in general, the outputs of the Co/Quad Analyzer are both phase and amplitude sensitive.

During most modal surveys, the level of the force input to the structure is controlled to a constant amplitude. If this is the case, then the simple product mode of the SD109B can be employed. The output term of interest under these conditions is generally FA sin θ. If a pure mode is excited and force and acceleration transducers used, the phase between the signals at resonance and the phase at each of the points of maximum modal response will be (+) or (–) 90 degrees. Ideally, the Co component of the output will be zero. As a practical matter, this may not always be the case since it is not always possible to excite a pure mode. It is extremely helpful when measuring a mode shape to assure that one is locked on to the true resonant condition.

If the Co/Quad Analyzer is used in conjunction with a Mechanical Impedance System, such as the SD1002E-1, a resonant dwell control feature is included. Figure 2-4 shows a typical configuration involving an optimum "3-Channel" measurement system for performing a modal analysis employing both force control and resonant dwell control. Under these conditions, the outputs of the impedance head are applied to a mechanical impedance measuring system, which is being used primarily for resonant dwell. The conditioned force channel signal is also used as one input to the Co/Quad Analyzer. Since the SD1002E-1 system is being used in the mechanical impedance mode, the filtered 100 kHz force signal is held at a constant level for two reasons; because the SD105C Amplitude Servo/Monitor is controlling at a constant level; and because of the loop action of the transfer function analyzer.

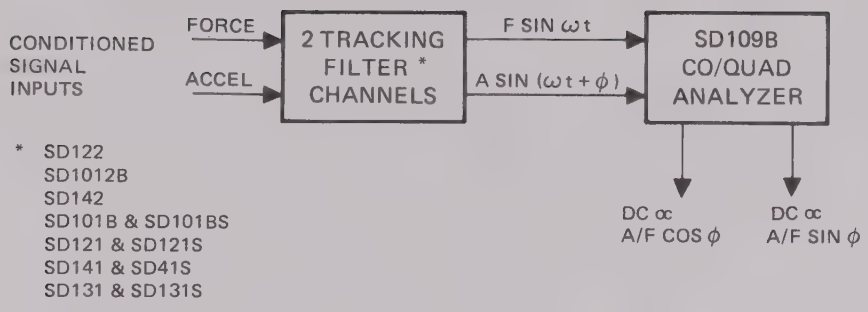


Figure 2-3. Basic Measurement Configuration for Vector Technique

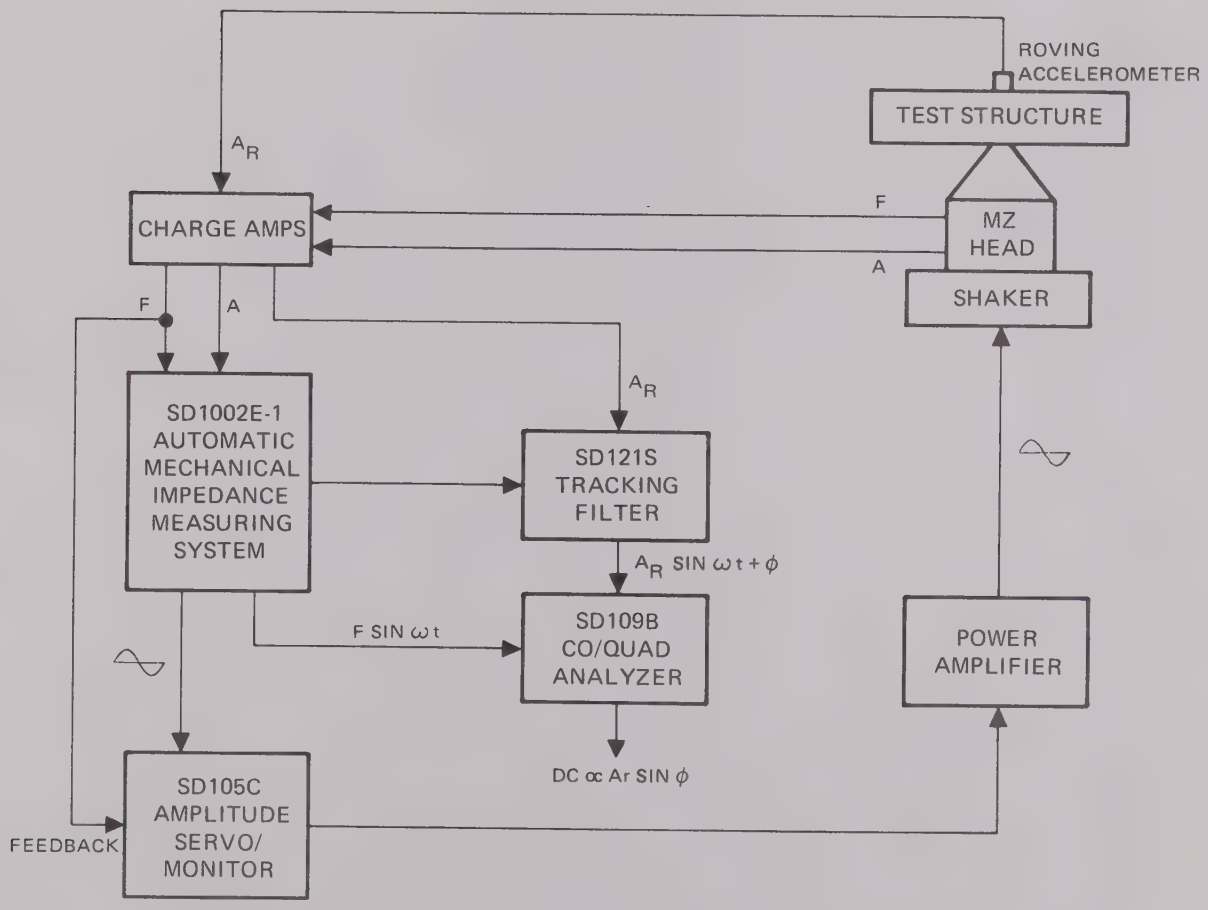


Figure 2-4. Modal Analysis with Resonant Dwell and Force Control

If servo control is not used, or if for some reason the transfer function analyzer is not used in the closed loop, Mechanical Impedance mode, it will probably be best to use the SD109B in the Ratio mode. In this mode, the signal corresponding to the filtered roving accelerometer will be applied to the A Channel input of the SD109B and the force signal applied to the B Channel input. As was pointed out above, the computed outputs of the Co/Quad Analyzer are sensitive to variations of either the amplitude of the two signal inputs, or of the phase difference between them. In the Ratio mode, variations of the B Channel input are minimized and a true ratio is displayed. Slightly more dynamic range is available from the Co/Quad Analyzer when it is used in the Product mode, compared to its use in the Ratio mode, and this may be a consideration if the analog outputs of the SD109B are to be digitized or applied to a digital voltmeter. In general, whether the Ratio or the Product mode of the Co/Quad Analyzer is used, the information describing the normalized mode shape is contained in the Quadrature channel output.

2.3.5 Procedure for Modal Analysis

Assume that the 3-Channel configuration of Figure 2-4 is to be employed to determine the mode shape of a structure using a roving accelerometer. The two tracking filter channels which will be driving the Co/Quad Analyzer should first be set into a full-scale internal calibration mode. If SD121 and SD122 tracking filters are employed, the calibration signal is obtained by placing the tracking filters in the Cal mode and placing the Carrier Generator into the Tuning mode. Assuming a full-scale signal indication on the meters of each of the respective tracking filter channels, the SD109B input attenuators are then set to read just below full scale. First, place the Co/Quad Analyzer in the Square mode of operation. Typically, either the 0 dB position or +5 dB position of the Co/Quad Analyzer can be used when a solid state tracking filter is employed. Adjust the full scale CAL screwdriver adjustment for each channel on the front of the SD109B for exactly a full scale reading on both the CO and QUAD meters. Next, place the Co/Quad Analyzer in the PRODuct mode. If the tracking filters are adjusted for zero phase shift between them, a full scale CO and zero QUAD indication should be seen. If for some reason a quadrature indication other than zero is obtained, adjust the phase on either of the tracking filter channels until the quadrature reading is zero. It is then a good practice to go back to the Square mode and ensure that the amplitudes of the individual I.F. signals have not varied appreciably because of the phase adjustment. If the reading on one or both channels of the SD109B is no longer full scale, adjust the FULL SCALE CAL potentiometer for a full scale indication. Note that adjusting the phase between tracking filter channels does have a tendency to vary the amplitude of the corresponding 100 kHz output. However, adjusting the amplitude calibration of the SD109B attenuator has little or no effect on the relative phase between channels. As a final calibration check and to calibrate an output readout, such as an X-Y plotter or Data Logger such as the SD133 Dynamic Data Terminal, the 3

CAL modes on the SD109B mode selector should be employed. With full scale calibration signals applied to the SD109B, the ZERO, CO and QUAD CAL modes furnish simple full-scale outputs to the external recording device.

If a roving accelerometer is being employed, it is good practice to test different points on the object being excited at resonance to assure that none of the readings obtained will be greater than full scale. It is also important to realize that as the pickup is moved from point to point, quite often a shock will be introduced into the charge amplifier driving a tracking filter. It is good practice to wait several seconds until the readings on the charge amplifier, tracking filter and Co/Quad Analyzer settle down. For sinusoidal type excitation, a minimum time constant can be employed in the Co/Quad Analyzer – typically, 0.1 second in each channel. The tracking filters and Co/Quad Analyzer can then be placed in the OPERATE mode and data taking can begin.

Figure 2-5 shows a typical frequency response recording in terms of magnitude and phase. The same data in terms of real and imaginary components is shown in Figure 2-6. Finally, Figure 2-7 shows the same vector components plotted versus one another giving the familiar Nyquist-type of display. To perform a modal analysis, the structure would be excited only at the resonant frequency and the roving pickup moved from point to point, providing a spatial reference. At each new location the magnitude and direction of the quadrature channel output is recorded and is then used as a direct indication of modal deformation. Since sinusoidal excitation is employed, it is a simple matter to calculate the equivalent displacement (D) from the measured acceleration (G) by the equation $G = 0.0511F^2D$.

where: G = peak measured accel (in/sec^2)
F = frequency of excitation (Hz)
D = peak to peak displacement (in.)

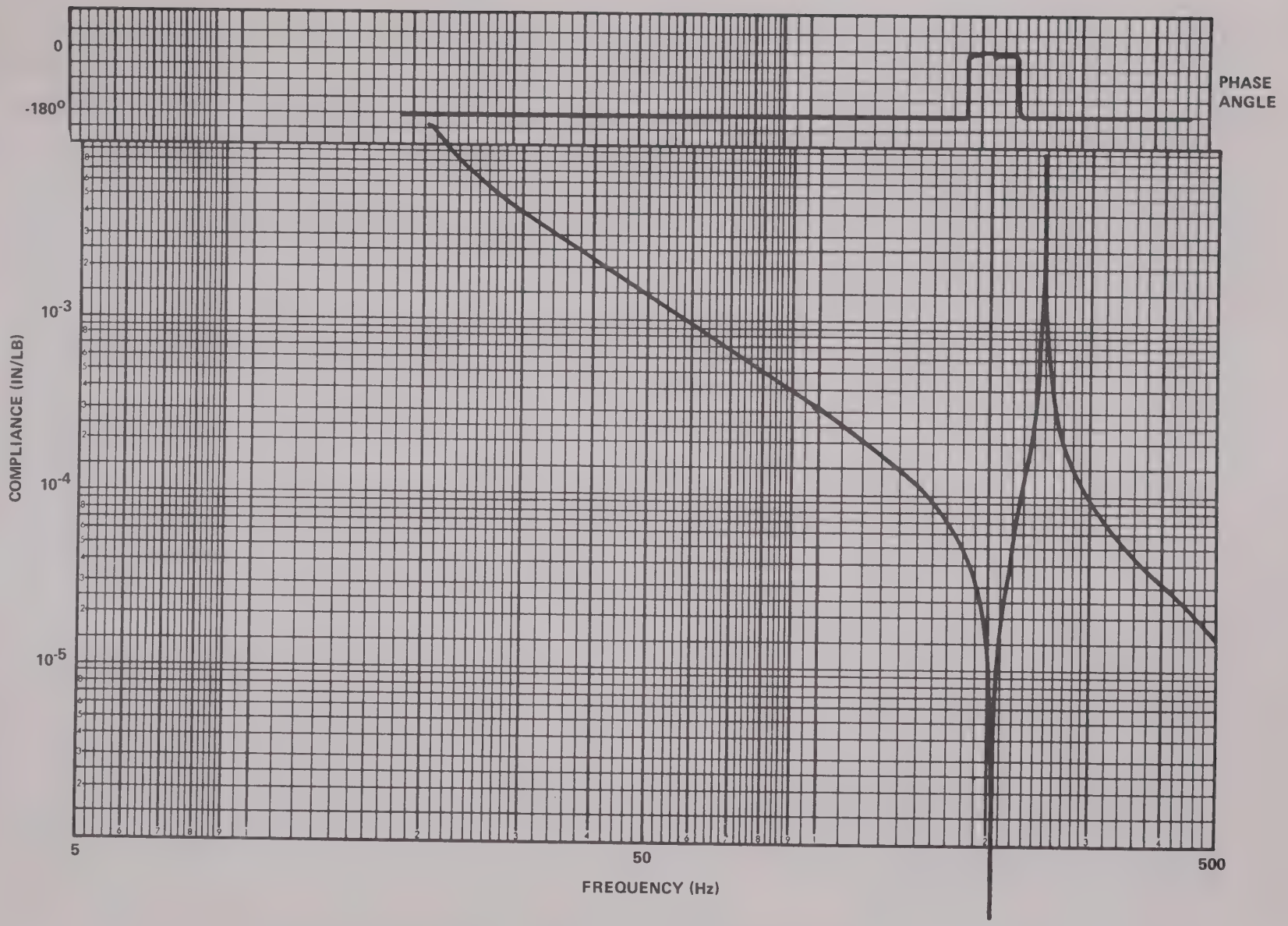


Figure 2-5. Typical Frequency Response Recording in Terms of Magnitude and Phase

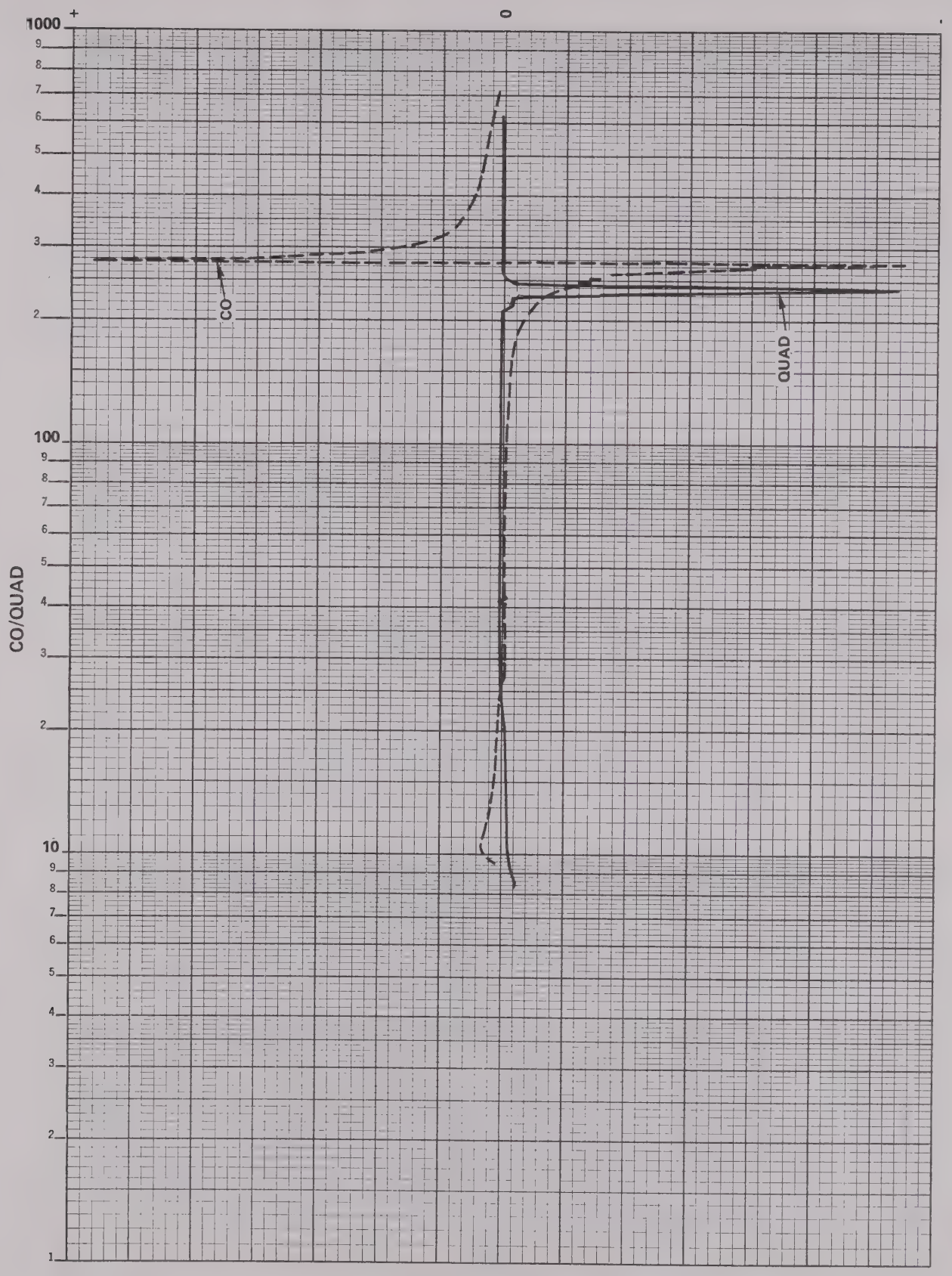


Figure 2-6. Typical Frequency Response Recording in Terms of Real and Imaginary Components

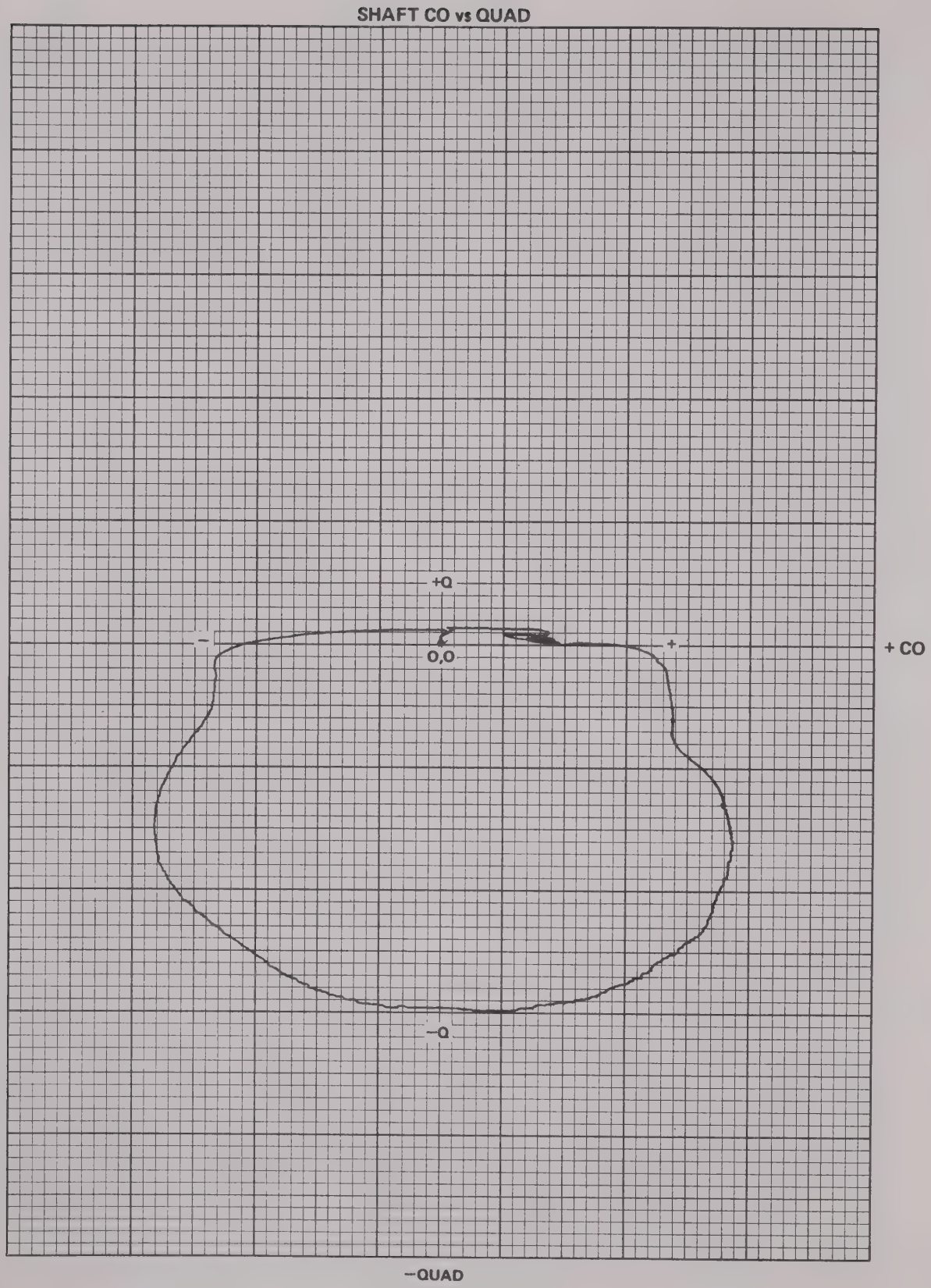


Figure 2-7. Plot of Vector Components — Nyquist-Type Display

SECTION III

CONCEPT AND THEORY OF OPERATION

3.1 CONCEPT OF OPERATION

The Model SD109B was designed specifically to operate in a system with SDC Dynamic Analyzer and/or Tracking Filter units. These units are capable of supplying to the SD109B the two 100-kHz filtered input signals. These signals are amplitude and phase coherent with original audio data signals. Because these are filtered 100-kHz signals, they may be referred to as:

$$A \sin(\omega_1 t + \theta) \text{ and } B \sin(\omega_1 t)$$

Within the SD109B, coincident and quadrature products are formed by first phase-shifting the B INPUT signal 90 degrees, thus forming the term:

$$B \cos(\omega_1 t)$$

When B cos ($\omega_1 t$) is multiplied by A sin ($\omega_1 t + \theta$), the quad product formed is:

$$\frac{AB}{2} [\sin(2\omega_1 t + \theta) + \sin\theta] \quad (1)$$

A low-pass filter removes the twice-frequency term, and the remaining dc is both amplitude and phase sensitive, varying between normalized limits. This term is the quadrature relationship between inputs A and B. Simultaneously, multiplication of the two signals without phase shift is performed, thus producing the co-products:

$$\frac{AB}{2} [-\cos(2\omega_1 t + \theta) + \cos\theta] \quad (2)$$

A low-pass filter removes the twice-frequency term, and the remaining dc is both amplitude and phase sensitive, varying between normalized limits. This term is the co-relationship between inputs A and B. The absolute product is obtained by rectifying the twice-frequency term of the above equation (2).

3.2 BLOCK DIAGRAM DESCRIPTION (See Figure 3-1.)

The 100-kHz filtered A INPUT signal, A sin ($\omega_1 t + \theta$), containing the amplitude and phase information of the original A audio signal, is routed through the selected setting of the A INPUT attenuator and applied to the A input amplifier. The amplified A signal is applied to the front-panel Mode switch where it is distributed to the CO or QUAD channel as selected.

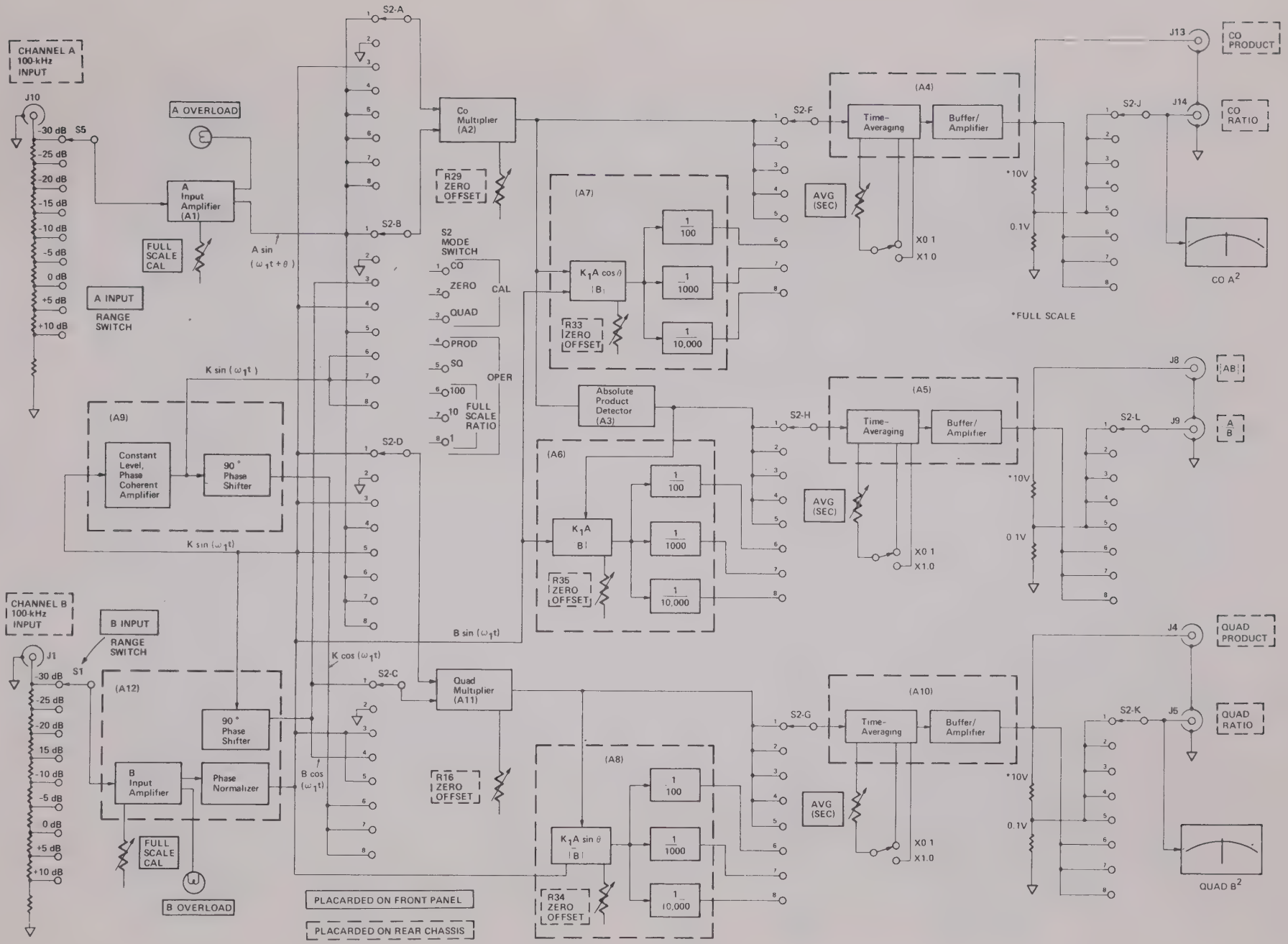


Figure 3-1. Functional Block Diagram

In like manner, the 100-kHz filtered B SIGNAL, B sin ($\omega_1 t$), is routed through the B INPUT attenuator, B input amplifier, and the Mode switch for distribution to the CO or QUAD channel as selected. A phase normalizing circuit in the B input amplifier facilitates calibrating for zero phase shift between A and B signals with a common signal applied to both the A and B INPUTS. The amplified B signal is shifted in phase 90 degrees to obtain B cos ($\omega_1 t$), and also amplified in a phase coherent constant-level amplifier yielding K sin ($\omega_1 t$); this signal is maintained at a constant level regardless of changes in the input signal level. K sin ($\omega_1 t$) is also phase shifted 90 degrees to produce K cos ($\omega_1 t$).

All of the signals described above are applied to the front-panel Mode switch for selection and distribution in proper pairs to the CO and QUAD Channel multipliers. The three OPERATE modes, PRODUCT, SQUARE, and RATIO, are described in the following paragraphs.

PRODUCT Mode:

With the Mode switch set to OPER PROD, signals A sin ($\omega_1 t + \theta$) and B sin ($\omega_1 t$) are applied to the CO multiplier, A2. The output of the CO multiplier is:

$$[\, A \sin(\omega_1 t + \theta) \,] \times [\, B \sin(\omega_1 t) \,]$$

$$= \frac{AB}{2} [\, \cos\theta - \cos(2\omega_1 t + \theta) \,] \qquad (3)$$

The high frequency component is filtered leaving the d-c component $\frac{AB}{2} \cos\theta$. This dc is amplified and routed to J13, CO PRODUCT OUTPUT, and to the CO-A^2 meter circuit. The output is calibrated to supply 10V full scale when both inputs A and B are in phase and are full scale; in this condition, the CO-A^2 meter reads +10. A variable time-averaging circuit at the input of the CO Channel d-c amplifier provides 0.1 to 10 seconds time averaging as controlled by the front-panel CO AVG (SEC) duo-dial control and multiplier switch.

The output of the CO multiplier is also routed to the product detector at the input of the ABSOLUTE Channel. Here, the twice-frequency term of equation (3) is detected (rectified), time-averaged, d-c amplified, and routed to J8, |AB| PRODUCT OUTPUT. The output is calibrated to supply 10V full scale when both inputs A and B are full scale; this output is not affected by any phase difference between the A and B inputs. A variable time-averaging circuit at the input of the ABSOLUTE Channel d-c amplifier provides 0.1 to 10 seconds time averaging of the output as controlled by the front-panel ABS AVG (SEC) duo-dial control and multiplier switch.

Also in the PRODUCT mode, signals A sin $(\omega_1 t + \theta)$ and B cos $(\omega_1 t)$ are applied to the QUAD Channel multiplier, A11. The output of the QUAD multiplier is:

$$A \sin(\omega_1 t + \theta) \times B \cos(\omega_1 t)$$

$$= \frac{AB}{2} [\sin\theta + \sin(2\omega_1 t + \theta)] \quad (4)$$

In like manner as previously described for the CO Channel, the d-c component $\frac{AB}{2} \sin\theta$ is extracted, amplified, and then routed to J14, QUAD PRODUCT OUTPUT, and to the QUAD-B^2 meter circuit. The output is calibrated to supply 10V full scale when both A and B inputs are full scale, and B lags A by 90 degrees; in this condition, the QUAD-B^2 meter reads +10. A variable time-averaging circuit at the input of the QUAD Channel d-c amplifier provides 0.1 to 10 seconds time averaging as controlled by the front-panel QUAD AVG (SEC) duo-dial and multiplier switch.

The following Table 3-1 lists the CO and QUAD meter readings and d-c output voltages for 0°, 90°, 180°, and 270° phase relationships between input signals A and B; input B lags input A. For the outputs given, the A and B input signal levels are full-scale.

Table 3-1. CO and QUAD Channel Meter Readings and D-C Outputs

Phase Difference Input B Lagging Input A	Meter		D-C Output (Volts)		
	CO-A^2	QUAD-B^2	CO	QUAD	ABSOLUTE
0°	+10	0	+10	0	10
90°	0	+10	0	+10	10
180°	-10	0	-10	0	10
270°	0	−10	0	−10	10

SQUARE Mode:

With the Mode switch set to OPER SQ, both inputs to the CO multiplier are A sin ($\omega_1 t + \theta$). The multiplier output is:

$$\frac{A^2}{2} [1 - \cos(2\omega_1 t + 2\theta)] \quad (5)$$

Both inputs to the QUAD multiplier are B sin ($\omega_1 t$). The multiplier output is:

$$\frac{B^2}{2} [1 - \cos(2\omega_1 t)] \quad (6)$$

As previously described for the PRODUCT mode, the d-c terms are extracted, time averaged, amplified, and routed to their respective meter circuits and rear-chassis output connectors. In the SQUARE mode, the d-c outputs are provided at PRODUCT OUTPUT connectors J13 (CO), J4 (QUAD), and J8 |AB| . Each is calibrated to provide 10V full scale when both A and B inputs are full scale; also in this condition, both front-panel meters read +10.

RATIO Mode:

With the Mode switch set to any of the OPER FULL SCALE RATIO A/B positions (1, 10, or 100), the inputs to the CO multiplier are A sin ($\omega_1 t + \theta$) and K sin ($\omega_1 t$). The output is therefore:

$$A \sin(\omega_1 t + \theta) \times K \sin(\omega_1 t)$$

$$= \frac{KA}{2} [\cos\theta - \cos(2\omega_1 t + \theta)]$$

$$= K_1 A [\cos\theta - \cos(2\omega_1 t + \theta)] \quad (7)$$

The d-c component of (7), $K_1 A \cos\theta$, is extracted and is applied along with B sin ($\omega_1 t$) to a servo loop. This circuit senses the amplitude of the B input signal and produces a d-c output that is proportional to 1/B. Figure 3-2 shows a simplified schematic diagram of the servo loop; a brief description follows:

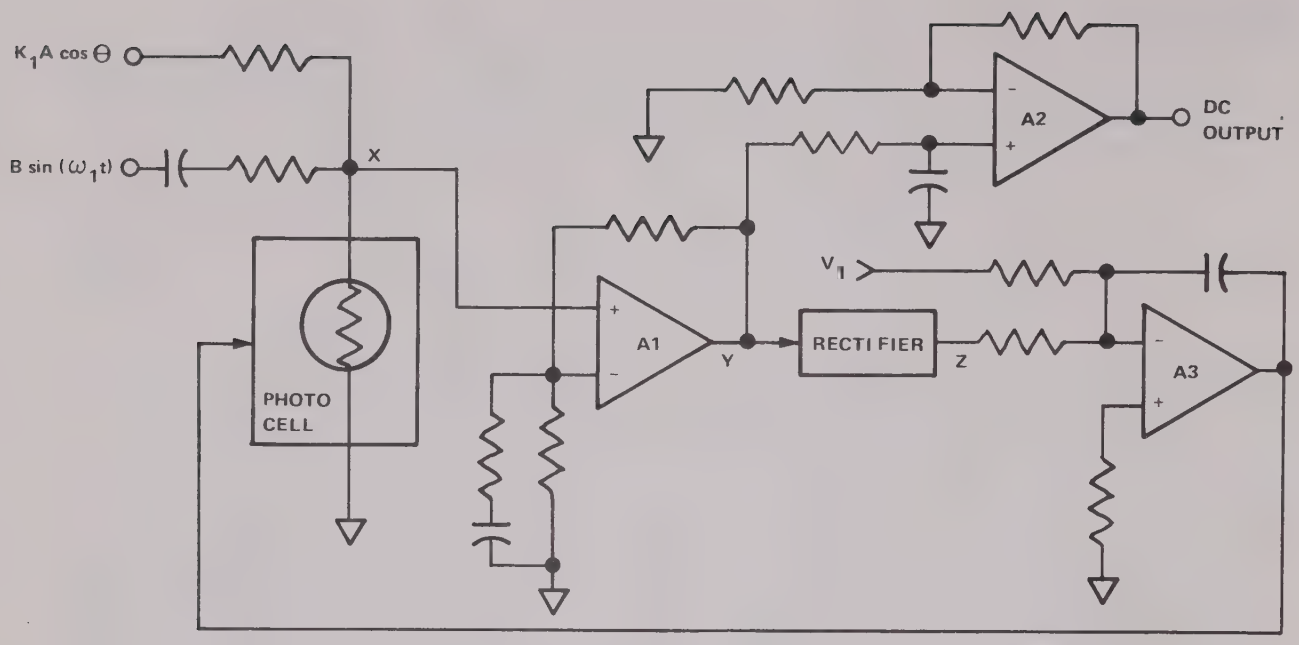


Figure 3-2. Simplified Schematic of Servo Loop

Referring to Figure 3-2, the d-c at point Z is held at some constant negative potential by varying the attenuation of the photo cell shunted across the input at point X. This implies that the output of amplifier A1 at point Y is also at a constant level. Because the a-c gain of amplifier A1 is constant, an inverse relation at point X is achieved; that is, for maximum B signal input, the photo cell is caused to have minimum resistance (maximum signal attenuation). This condition is mathematically described as follows:

$$\text{dc at point X} = \frac{K_1 A \cos\theta}{K_2 \dfrac{B}{B_{\text{full scale}}}} \text{, where } K_2 > \frac{B_{\text{full scale}}}{B_{\text{minimum}}}$$

Because $B_{\text{full scale}}$ is constant,

$$\text{dc at point X} = \frac{K_3 A \cos\theta}{B}$$

The dc at point X is buffered and amplified by A1. The a-c component at Y is filtered, and the d-c component is amplified and normalized by A2 to provide 100 mV d-c full-scale output for each range. The constant K_3 is determined so that with in-phase A and B inputs applied, a 100

mV full-scale signal is available at the CO RATIO OUTPUT, J14, when the following conditions exist: A = B in RATIO 1 position, B = 0.1A in RATIO 10 position, and B = 0.01A in RATIO 100 position. The output (A/B) $\cos\theta$ is also continuously displayed on the CO-A^2 meter.

Similarly, (A/B) $\sin\theta$ is available as 100 mV full-scale signal at the QUAD RATIO OUTPUT, J5, and is continuously displayed on the QUAD-B^2 meter. The absolute ratio $|A/B|$ is available at J4 as 100 mV full scale.

Note that for valid results when operating in the RATIO 1 mode position, the B input signal must be equal to or greater than 0.1 of the selected full-scale range. Also, for RATIO 10 and 100, the B input signal must be equal to or greater than 0.01 B full scale. However, the A input signal can be any value from zero to full scale.

SECTION IV

CALIBRATION AND MAINTENANCE

4.1 GENERAL

Before calibrating the instrument, the technician is advised to study carefully the theory of operation in Section III of this manual. Refer also to the front-panel and rear-chassis descriptions given in Section I.

NOTE

Figure 4-1 is a top view of the instrument showing calibration adjustments. A layout drawing showing the location of the circuit cards is located on the inside surface of the top cover.

Throughout the calibration procedure, calibration controls and parts on a circuit card are referenced by their circuit card and part number alphanumeric designator. For example, on Power Supply circuit card A13, R24 is referred to as A13R24.

4.2 CALIBRATION PROCEDURE

CAUTIONS

1) To ensure proper calibration of the instrument, the following procedure must be performed in its entirety and in the sequence listed.

2) Do not extend printed circuit cards during the calibration procedure. All calibration controls and pertinent test points are accessible at the top of each circuit card.

Table 4-1 lists the test equipment required; except where noted, test equipment having equal or better operational characteristics may be substituted.

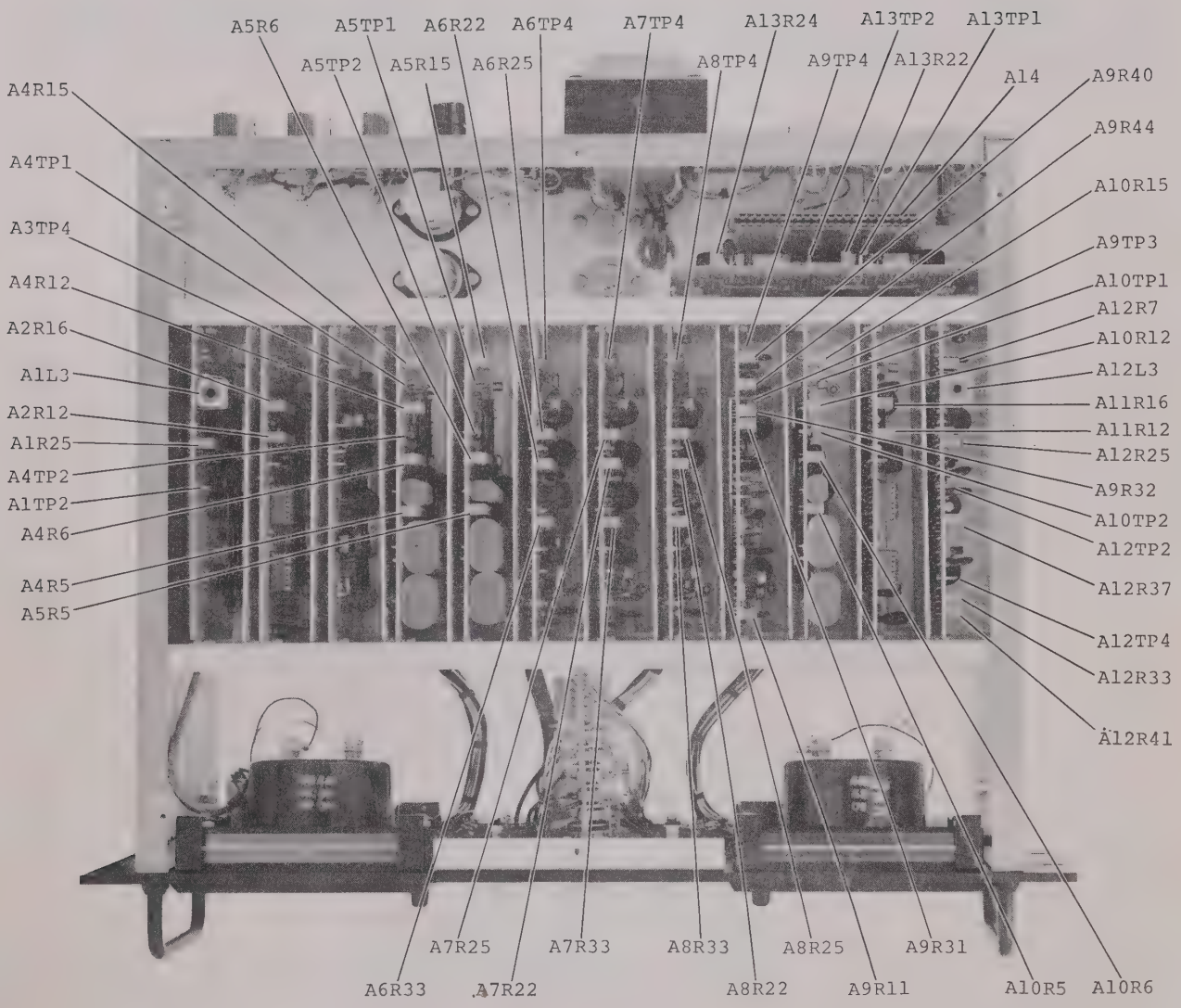


Figure 4-1. Top Chassis View

Table 4-1. Test Equipment Required

Name	Model No.	Manufacturer
Attenuator	350D	Hewlett-Packard
Digital Voltmeter	3440A	Hewlett-Packard
Frequency Counter	5223L	Hewlett-Packard
Oscilloscope	545	Tektronix
Phase Meter	SD110	Spectral Dynamics Corp.
RMS Voltmeter	3400A	Hewlett-Packard
100-kHz signal sources (2 required)	SD101A,B, SD1012,B	Spectral Dynamics Corp.
Oscillator	4200	Krohnhite
Sweep Oscillator	SD104	Spectral Dynamics Corp.

4.2.1 Preliminary Setup

a. With the power off, check that the 115/230 switch on the rear chassis is set properly and that the correct value of fuse is installed.

b. Check that both front-panel meters are at zero; mechanically adjust if necessary.

c. Remove instrument top cover.

d. Adjust the CO, QUAD, and ABS AVG (SEC) time averaging controls for 0.1 second (multiplier switch at 0.1 and duo-dial at 1). Set both Input Range switches to 0 dB position.

e. Connect power cord to primary power source, and press the POWER push button/indicator; it will glow red indicating power is on. Allow at least a 1-hour warmup.

4.2.2 Power Supply (A13)

a. Connect DVM to A13TP2; adjust A13R24, if necessary, for -15V (-14.980 to -15.020V).

b. Connect DVM to A13TP1; adjust A13R22, if necessary, for +15V (+14.980 to +15.020V).

c. With oscilloscope, check ripple at A13TP2 and A13TP1. Ripple should be less than 3 mV p-p.

4.2.3 Multipliers (Circuit Cards A2 and A11)

NOTE

The integrated circuit amplifiers A1 and A2 on circuit cards A2 and A11 (Multiplier Assembly 11956-1) are factory-selected matched pairs in order to ensure proper operation of their respective circuit cards. Therefore, A1 and A2 should not be replaced in the field by standard "off-the-shelf" or even preselected components. Failure of these components makes it mandatory that the circuit cards be returned to the factory for repair and calibration. The following (steps a. through f.) is the procedure for checking the calibration of circuit card A2 or A11. However, note that it is accomplished for either card using the receptacle normally occupied by circuit card A2. Actual calibration of a circuit card, involving steps g. through o. is NOT recommended if satisfactory results are obtained from the calibration check procedure (a. through f.).

CALIBRATION CHECK — Circuit Card A2 or A11 (In the A2 receptacle)

a. Set the front-panel mode selector to the CAL ZERO position. On the rear of the chassis, with a digital voltmeter at J13 (CO PRODUCT OUTPUT), adjust R29 for zero (−2.5 to +2.5 mVdc).

b. Switch the front-panel mode selector to the CAL CO position and the A INPUT and B INPUT range switches to the 0 dB position.

Apply a common 1 Vrms, 100 kHz (99.99 to 100.01 kHz) signal to connectors J1 and J10 (CHANNEL A and CHANNEL B INPUTS) on the rear of the chassis.

Adjust the front panel FULL SCALE CAL control for a full-scale indication (+10) on the CO-A^2 meter.

c. Switch the front-panel mode selector to the CAL QUAD position and adjust the FULL SCALE CAL control for a full-scale indication (+10) on the QUAD-B^2 meter.

d. Switch the front-panel mode selector to the OPERATE PROD position; on the rear of the chassis remove the input to J10 (CHANNEL A) and short J10 inner conductor to ground. With a digital voltmeter at J13 (CO PRODUCT OUTPUT), note the DVM indication as "X".

e. Remove the short to ground at J10 and reconnect input signal. Remove the input to J1 (CHANNEL B) and short J1 inner conductor to ground. Note the DVM indication at J13 as "Y". -4.16

f. If the difference between "X" and "Y", as noted in the preceding steps d. and e. is less than 20 mV, and the absolute value of "A" or "Y" individually is less than 20 mV, the circuit card being checked (either A2 or A11) meets all calibration requirements; DO NOT ATTEMPT FURTHER CALIBRATION. If circuit cards A2 or A11 do not meet the above calibration requirements, proceed with the following calibration procedure for either card installed in the A2 receptacle.

CALIBRATION OF CIRCUIT CARD A2 or A11 (In the A2 receptacle)

g. On the rear of the chassis, short to ground the inner conductor of J10. With a full-scale input (1 Vrms, 100 kHz) applied to J1, measure the output at J13 with a DVM and note the indication as "C".

h. Remove the full-scale input to J1 and short its inner conductor to ground. Note the DVM measurement at J13 as "D".

i. If the difference between the "C" and "D" measurements noted in the two preceding steps is less than or equal to 10 mV, go to step j. that follows; if greater than 10 mV go to step l.

j. Short to ground the inner conductor to J1. With a full-scale input applied to J10 (1 Vrms, 100 kHz), measure the output at J13 with a DVM and note the indication as "C".

k. Remove the full-scale input to J10 and short its inner conductor to ground. Note the DVM measurement at J13 as "D".

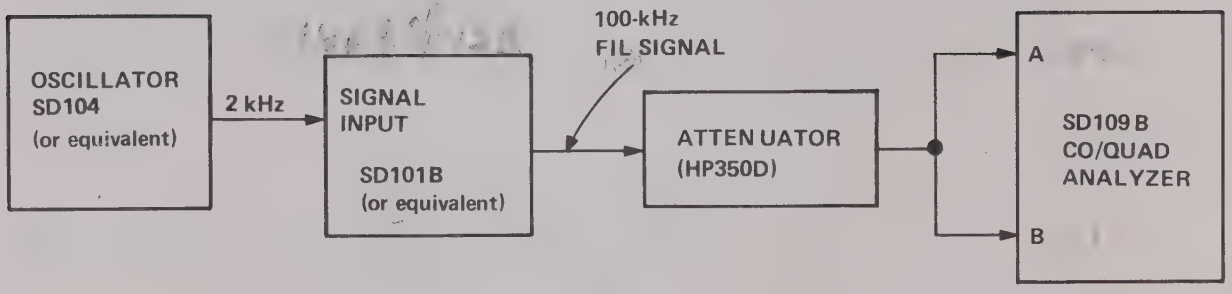
l. Adjust A2R16 (A11R16) as necessary to minimize the difference between "C" and "D". (Repeat the preceding steps g. through l., as applicable, until the difference between "C" and "D" is minimized to the fullest.)

m. Connect an oscilloscope to pin 21 of the card being calibrated (A2 or A11 in the A2 receptacle) and apply a common full-scale (100 kHz) signal to connectors J1 and J10. Adjust A2R12 (A11R12) using the limits of its full range as necessary to obtain the best possible symmetry of the waveform.

n. Install circuit cards A2 and A11 in their receptacles, set the front-panel mode selector to the CAL ZERO position and adjust R16 and R29 on the rear of the chassis for zero (−2.5 to +2.5 mV) at connectors J4 and J13, respectively.

4.2.4 Input Amplifier (A1 and A12)

a. Connect SD109B in test setup shown below:



In the above test setup, apply a common 100-kHz, 1 Vrms (0.995 to 1.005 Vrms), signal to both CHANNEL A and CHANNEL B 100-kHz INPUTS (J10 and J1). The recommended compatible input signal is the 100-kHz FILTERED SIGNAL OUTPUT of the SD101A, SD101B, SD1012, SD1012B, etc. Dynamic Analyzer/Tracking Filter. The SIGNAL INPUT to the unit being used should be a sine wave at approximately 2 kHz. Set Mode switch on the SD109B to the OPERATE SQ position.

b. Connect RMS voltmeter to A1TP2; adjust A1L3 for maximum reading.

c. Adjust front-panel CO FULL SCALE CAL for 0.95 Vrms (0.94 to 0.96 Vrms).

d. Adjust A1R25 until the A OVERLOAD indicator lamp just starts to glow.

e. Now adjust CO FULL SCALE CAL for 0.85 Vrms (0.84 to 0.86 Vrms).

f. Connect RMS voltmeter to A12TP2; adjust A12L3 for maximum reading.

g. Adjust front-panel QUAD FULL SCALE CAL for 0.95 Vrms (0.94 to 0.96 Vrms).

h. Adjust A12R25 until the B OVERLOAD indicator lamp just starts to glow.

i. Now adjust QUAD FULL SCALE CAL for 0.85 Vrms (0.84 to 0.86 Vrms).

j. Connect RMS voltmeter to A12TP4. Adjust A12R33 for 0.85 Vrms (0.84 to 0.86 Vrms).

4.2.5 Product Averager (A4 and A10)

a. Set Mode switch to CAL ZERO. Check that 100-kHz common input to both CHANNEL A and B is 1 Vrms (0.995 to 1.005 Vrms).

b. Connect DVM to A4TP1; adjust ZERO OFFSET R29 (rear chassis) for minimum DVM reading, less than 0.2 mV.

c. Connect DVM to A4TP2; short A4TP1 to ground and adjust A4R5 for DVM reading of ±0.1 mV.

d. Remove short from A4TP1. Set Mode switch to CAL CO position, and adjust A4R6 for 10V (9.99 to 10.01V) reading on DVM at A4TP2.

e. Connect DVM to CO RATIO OUTPUT J14 (rear chassis). Adjust A4R15 for DVM reading of 100 mV (99.9 to 100.1 mV).

f. Adjust A4R12 for front-panel CO-A^2 meter reading of +10, full scale.

g. Set Mode switch to CAL ZERO.

h. Connect DVM to A10TP1; adjust ZERO OFFSET R16 (rear chassis) for minimum DVM reading, less than 0.2 mV.

i. Connect DVM to A10TP2; short A10TP1 to ground and adjust A10R5 for DVM reading of ±0.1 mV.

j. Remove short from A10TP1. Set Mode switch to CAL QUAD position, and adjust A10R6 for 10V (9.99 to 10.01V) reading on DVM at A10TP2.

k. Connect DVM to QUAD RATIO OUTPUT J5 (rear chassis). Adjust A10R15 for DVM reading of 100 mV (99.9 to 100.1 mV).

l. Adjust A10R12 for front-panel QUAD-B^2 meter reading of +10 full scale.

4.2.6 Product Averager (A5), Product Detector (A3)

a. Set Mode switch to CAL CO position and reduce common 100-kHz input by 30 dB.

b. Connect DVM to A3TP4; adjust ZERO OFFSET R15 (rear chassis) for reading of 1.4 mV (1.3 to 1.5 mV).

c. Short A5TP1 to ground. Connect DVM to A5TP2. Adjust A5R5 for reading of ±0.1 mV.

d. Remove short from A5TP1. Increase 100-kHz input by 30 dB; i.e., return to 1 Vrms.

e. Adjust A5R6 for 10V (9.99 to 10.01V) on DVM at A5TP2.

f. Connect DVM to |A/B| RATIO OUTPUT J9. Adjust A5R15 for reading of 100 mV (99.9 to 100.1 mV).

4.2.7 Input Amplifier (A12)

a. Set Mode switch to CAL CO position. Check that common A+B input is 1 Vrms.

b. Connect DVM to QUAD PRODUCT OUTPUT J4 (rear chassis); adjust A12R37 for 0V ±10 mV.

c. Set Mode switch to CAL QUAD.

d. Connect DVM to CO PRODUCT OUTPUT J13. Note value; then adjust A12R37 for DVM reading of one-half noted value.

e. Set Mode switch to OPERATE PRODUCT.

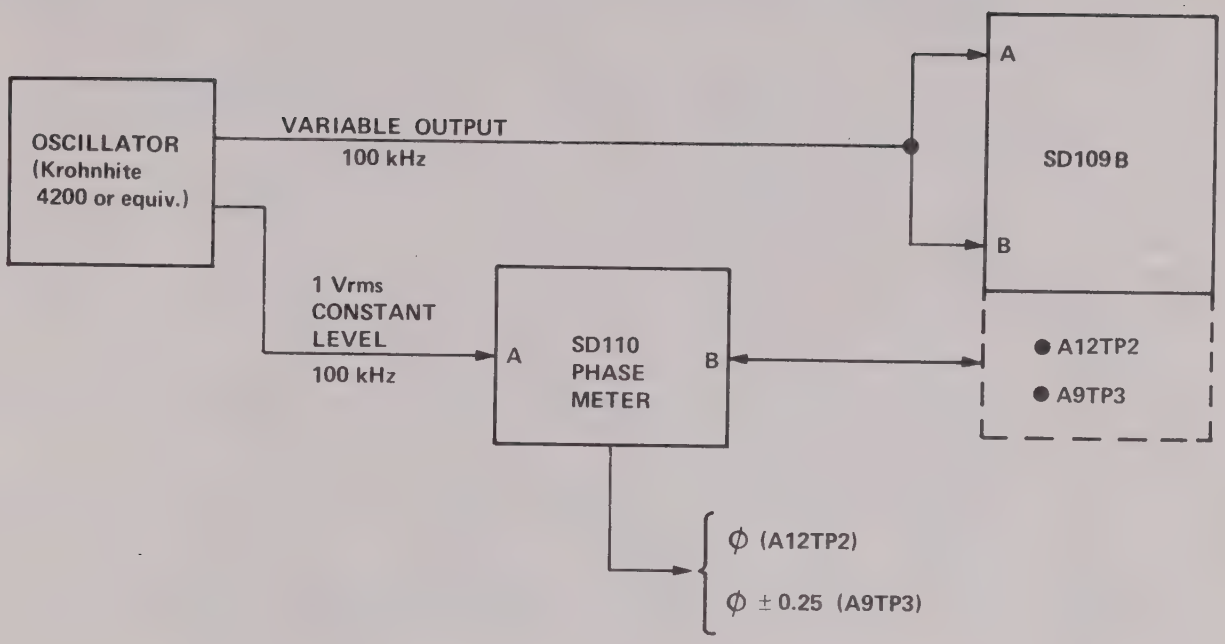
f. Connect DVM to QUAD PRODUCT OUTPUT J4. Adjust A12R7 for 0V ±10 mV.

4.2.8 ϕ Normalized Limiter (A9)

a. Set both INPUT A and B range switches to 0 dB position.

b. Replace common signal input source from Dynamic Analyzer/Tracking Filter with test oscillator as shown below.

Adjust oscillator for 100 kHz ±10 Hz, at 1 Vrms (0.995 to 1.005 Vrms).



c. Set Mode switch to OPERATE SQ.

d. Connect RMS voltmeter to A9TP3; adjust A9R32 for 0.85 Vrms (0.84 to 0.86 Vrms).

CAUTION

If an SD110 is used in the following steps, be sure S9, 20-kHz INPUT L.P. FILT switch (inside SD110), is set to OUT.

e. Measure phase difference between oscillator constant output and A12TP2. Record this reading as "ϕ".

f. Now measure phase difference between oscillator constant output and A9TP3. Adjust A9R31 for phase difference same as $\phi \pm 0.25°$.

g. Attenuate oscillator output by 50 dB. (Use front-panel attenuator on oscillator.)

h. Adjust A9R11 for minimum phase difference between oscillator constant output and A9TP3; should be $\phi \pm 10°$.

i. Check phase difference between common input and A9TP3 for each oscillator signal level listed below:

Oscillator Attenuator Setting	Phase Difference (ϕ value from step e.)
0 dB	$\phi \pm 0.25°$
-10 dB	$\phi \pm 0.5°$
-20 dB	$\phi \pm 0.5°$
-30 dB	$\phi \pm 1.0°$
-40 dB	$\phi \pm 1.5°$

Return oscillator attenuator to 0 dB, to supply 1 Vrms (0.995 to 1.005 Vrms) full-scale common input to A and B INPUTS.

j. Disconnect phase meter at A9TP3 and connect RMS voltmeter. Re-adjust A9R32 for 0.85 Vrms (0.84 to 0.86 Vrms).

k. Connect RMS voltmeter to A9TP4; adjust A9R40 for 0.85 Vrms (0.84 to 0.86 Vrms).

l. Disconnect RMS voltmeter at A9TP4.

m. Connect SD110 Phase Meter between A9TP3 and A9TP4. Adjust A9R44 for phase difference of 90° (89.75° to 90.25°) on the meter.

n. Connect RMS voltmeter to A9TP4; re-adjust A9R40 for reading of 0.85 Vrms (0.84 to 0.86 Vrms).

4.2.9 CO/QUAD Channel (A/B), (A6, A7, A8)

a. Set both A INPUT and B INPUT range switches to 0 dB (1 Vrms full scale). Set Mode switch to RATIO 100.

b. Apply 1 Vrms full scale, 100-kHz to CHANNEL B INPUT; short CHANNEL A INPUT.

c. Connect DVM to CO RATIO OUTPUT J14. Adjust R33 (below J14) for 0V ± 0.1 mV.

d. Set Mode switch to RATIO 10. Carefully fine-adjust R33 for 0V ± 0.1 mV at J14.

e. Return Mode switch to RATIO 100.
OF 0.5 mV BETWEEN RATIO 100 & 10:

f. Connect DVM to QUAD RATIO OUTPUT J5. Adjust R34 (below J5) for 0V ± 0.1 mV.

g. Set Mode switch to RATIO 10. Carefully fine-adjust R34 for 0V ± 0.1 mV at J5.

h. Return Mode switch to RATIO 100.

i. Connect DVM to |A/B| RATIO OUTPUT J9. Adjust R35 (below J9) for 0V ± 0.1 mV.

j. Set Mode switch to RATIO 10. Carefully fine-adjust R35 for 0V ± 0.1 mV at J9.

k. Return Mode switch to RATIO 100, and check that CHANNEL B INPUT is full scale.

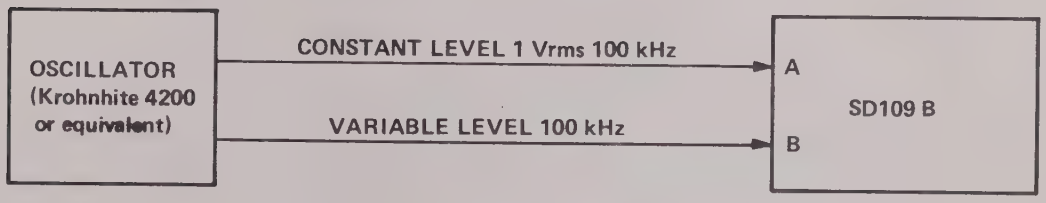
l. Monitor A7TP5 with oscilloscope; adjust A7R25 for 0.5V p-p (0.48V p-p to 0.52V p-p).

m. Monitor A8TP5 with oscilloscope; adjust A8R25 for 0.5V p-p (0.48V p-p to 0.52V p-p).

n. Monitor A6TP5 with oscilloscope; adjust A6R25 for 0.5V p-p (0.48V p-p to 0.52V p-p).

o. Reconnect inputs as shown below, with 100-kHz constant level 1 Vrms (0.995 to 1.005 Vrms) applied to CHANNEL A INPUT (J10), and variable output applied to CHANNEL B INPUT (J1).

Adjust B INPUT for 100 mV rms (99.5 to 100.5 mV rms). Ensure that A and B inputs are in-phase. (The recommended Krohnhite 4200 satisfies these requirements.)



p. Set Mode switch to RATIO 10.

q. Connect DVM to CO RATIO OUTPUT J14. Adjust A7R42 (on circuit card A7) for 99 mV (98 to 100 mV).

r. Connect DVM to A/B RATIO OUTPUT J9. Adjust A6R42 for 99 mV (98 to 100 mV).

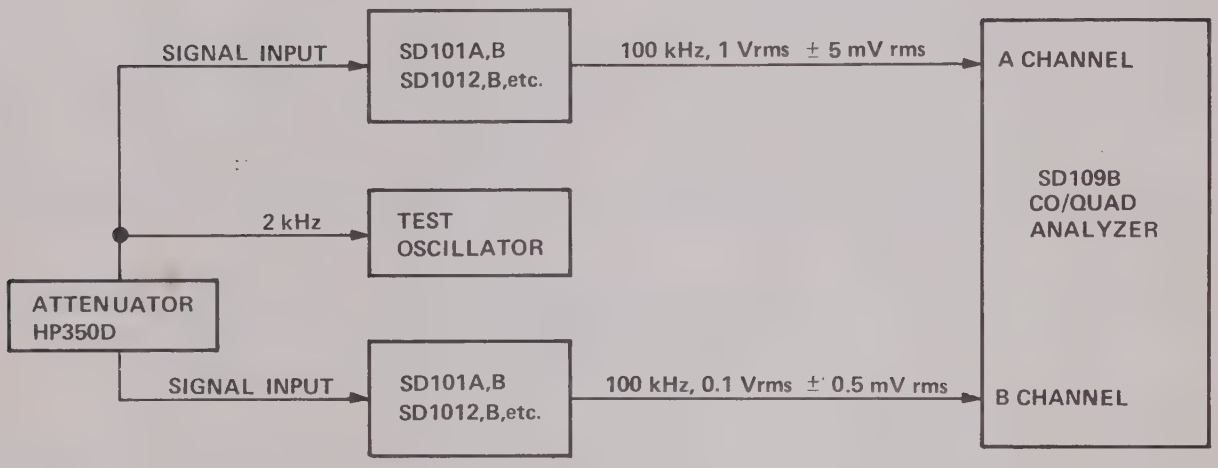
s. Set Mode switch to RATIO 100. Increase oscillator output attenuation by 20 dB; B CHANNEL input is now 0.01 Vrms.

t. Connect DVM to |A/B| RATIO OUTPUT J9. Adjust A6R22 for 100 mV (99.5 to 100.5 mV).

u. Connect DVM to CO RATIO OUTPUT J14. Adjust A7R22 for 100 mV (99.5 to 100.5 mV).

v. Apply a common 1 Vrms ±5 mV rms signal to A and B INPUTS. Connect DVM to CO RATIO OUTPUT, J14. Set Mode switch to RATIO 1 position. Adjust A12R41 for 99 mV (98 to 100 mV) at J14.

w. Replace test oscillator signal source with setup shown below:



Adjust A CHANNEL source to supply 1 Vrms (0.995 to 1.005 Vrms). Adjust B CHANNEL source to supply 100 mV rms (99.5 to 100.5 mV rms). Using front-panel PHASE ADJ on Dynamic Analyzer/Tracking Filter, adjust for a 90°±0.5°phase difference between CHANNEL A and CHANNEL B 100-kHz INPUTS throughout the following steps. (The recommended SD101A, B, etc. (or SD1012, B) units can be adjusted to provide these signal requirements.)

x. Set Mode switch to RATIO 10.

y. Connect DVM to QUAD RATIO OUTPUT J5. Adjust A8R42 (on circuit card A8) for 99 mV (98 to 100 mV).

z. Set Mode switch to RATIO 100. Using signal source level controls, reduce the B CHANNEL input by 20 dB, to 10 mV rms (9.95 to 10.05 mV rms).

aa. With DVM at J5, adjust A8R22 for 100 mV (99.5 to 100.5 mV).

Remove power, disconnect all test equipment replace the instrument top cover, and place instrument in service.

4.3 USING EXTENDER CARD

To aid in servicing the instrument, a circuit card extender (A14) is supplied (located in socket XA14 inside the chassis). Note that except for the extender card, each circuit card is individually keyed to its receptacle. Each card is slotted, and a "key" is located between two pins of each receptacle. Therefore, the "key" must be removed before the extender can be inserted.

CAUTION

Be sure power is off before removing or replacing circuit cards.

A layout drawing showing the location of the circuit cards is located on the inside surface of the top cover. On this drawing, the location of each circuit card "key" is denoted by its two adjacent pin numbers. For example, on circuit card A13, the "key" is located between pins 13 and 14.

CAUTION

To prevent possibility of plugging a circuit card in the wrong socket, be sure to replace the "keys" after servicing the instrument.

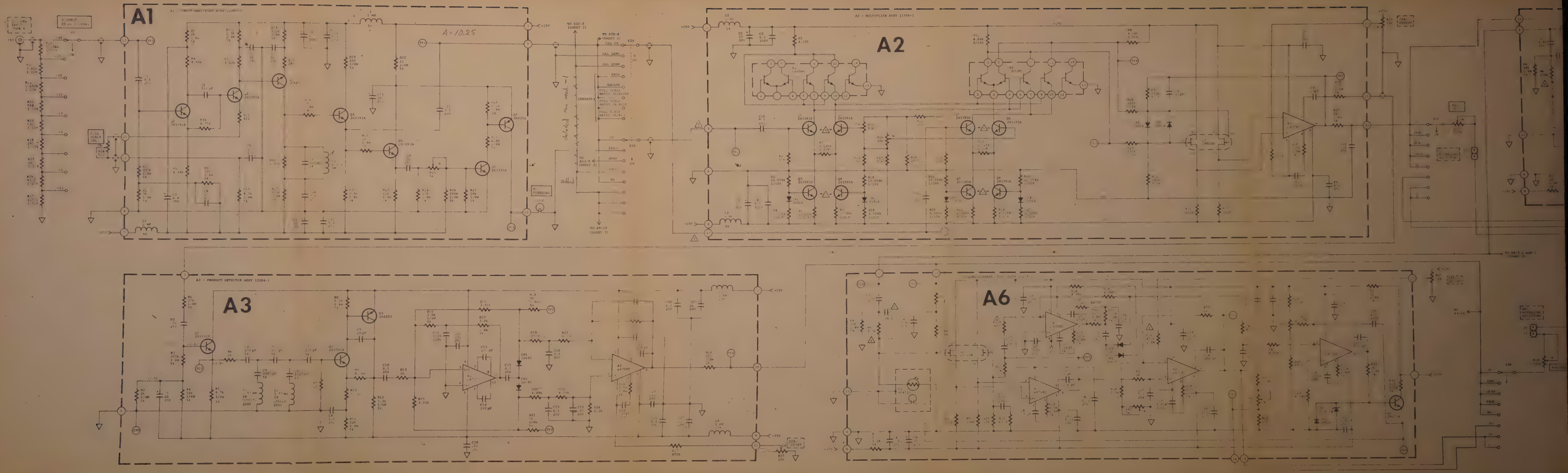
4.4 PARTS REPLACEMENT

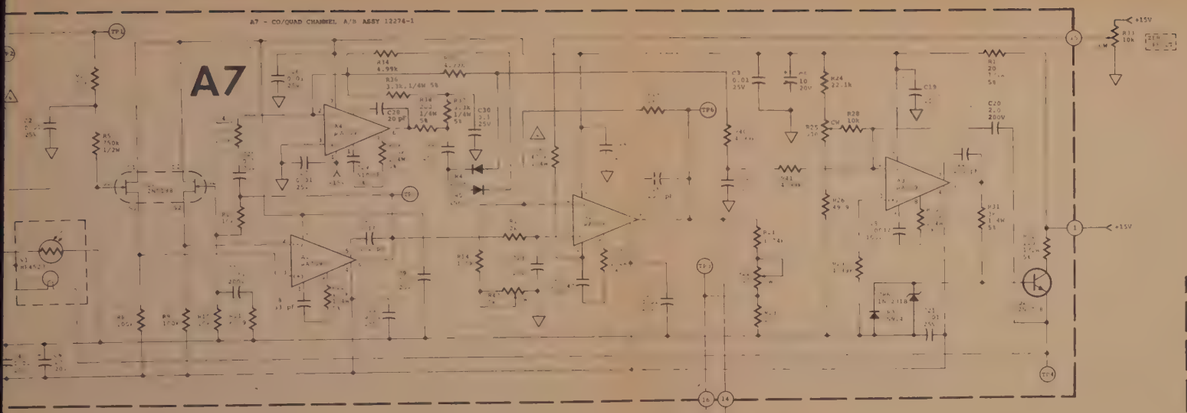
Except for some special SDC specification-controlled parts listed in Table 4-2, which must be ordered from Spectral Dynamics Corporation, standard electronic parts have been used throughout the instrument. Spectral Dynamics Corporation maintains a complete stock of all parts and complete printed circuit card assemblies used in the instrument.

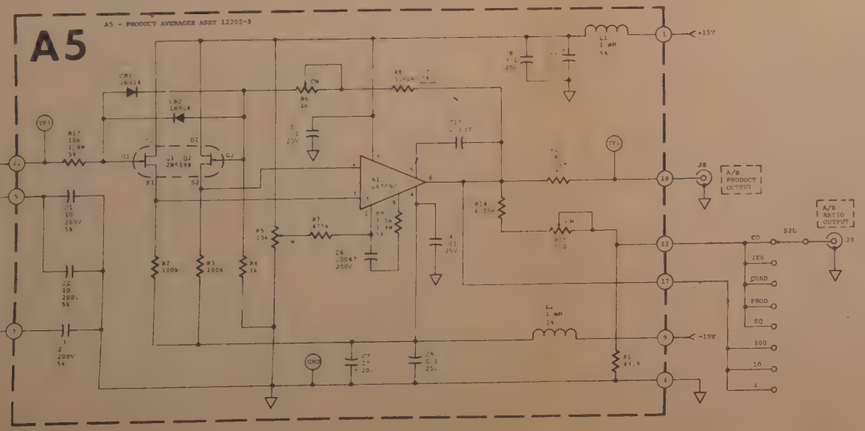
When ordering a replacement part or assembly, be sure to include the reference designation, the part number or manufacturers designation, and a complete description; include also the instrument name, model number, and serial number.

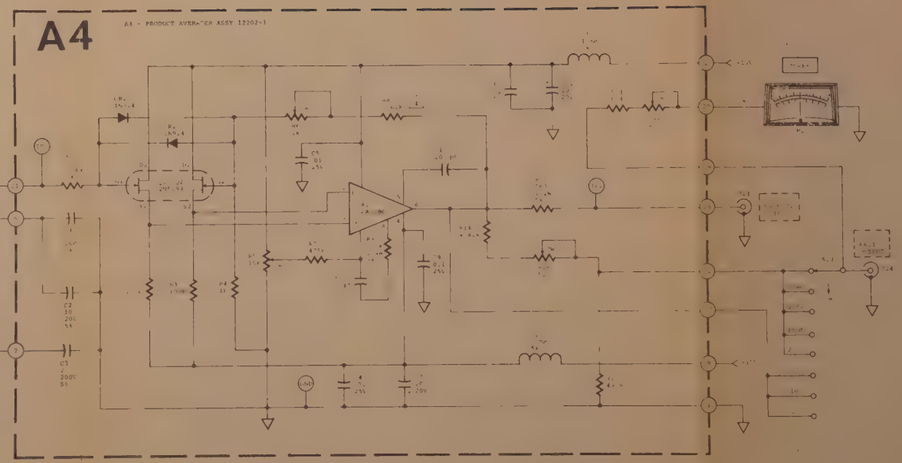
Table 4-2. Special Replaceable Parts List

Reference Designator	Description	Part Number Mfr. Designation
A1	Input Amplifier	12200-3
A2,A11	Multiplier	11956-1
A3	Product Detector	12204-1
A4,A10	Product Averager	12202-1
A5	Product Averager	12202-3
A6-A8	CO/QUAD Channel (AB)	12274-1
A9	ϕ Normalized Limiter	12336-1
A12	Input Amplifier	12200-1
A13	Power Supply	11628-3
A14	Extender	12348-1
A2Q1-A2Q8 & A11Q1-A11Q8	Transistor Pair, 2N3391A	20013-051
	Power Cord	Belden 17258-S
	Knob, A INPUT, B INPUT, Mode	20013-175
DS1,2	Lamp, A & B OVERLOAD	CF03-WTB-2102
DS3	Lamp, POWER Indicator	A1H
M1,M2	Meter, CO-A^2, QUAD-B^2	20013-136
S1,S5	Switch, Range, B INPUT, A INPUT	20013-309
S2A-L	Switch, Mode	20013-308
S3,S4,S6	Switch, Multiplier (X0.1,X1.0) AVG (SEC)	7101
S7	Switch, POWER	54-61681-26
S8	Switch, 115/230	46256LF
T1	Transformer, Power	20013-255


A1
A=10.25
A2
A3
A6







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METROLOGY LAB.
DO NOT REMOVE
FROM AREA

REVISIONS						
LTR	ZONE		DESCRIPTION	DATE	APPROVED	
A	—	—				
B	3 K5	5 4.7K	CHG A4: A7 AND A8 Y1 WAS HP4536, R4 WAS 249K. R25 WAS 200. R26 WAS 348. R32 WAS 49. ADDED (4) TO R33, R62, 5x WAS R33 EA. ON A3 AND A11 (7) WAS ADDED TO PINS 1 AND 15 AND TO BOTH.	L.S. 1-73		
C	K25	5 4.7K 5 4.7K	ON A5, A7 APC A8 Y1 WAS HP4529. R5 WAS 7.5K, R4 WAS 392K. C28 WAS 0.47	L.S. 1-73		
D	K25	R4 4.7K	ON A3 AND A11 0.15 WAS ADDED TO R28 THRU R31.	L.S. 1-73		
E	K25	R4 4.7K	CHG "4, A5 AND A10 4 WAS ADDED TO R8.	L.S. 1-73		

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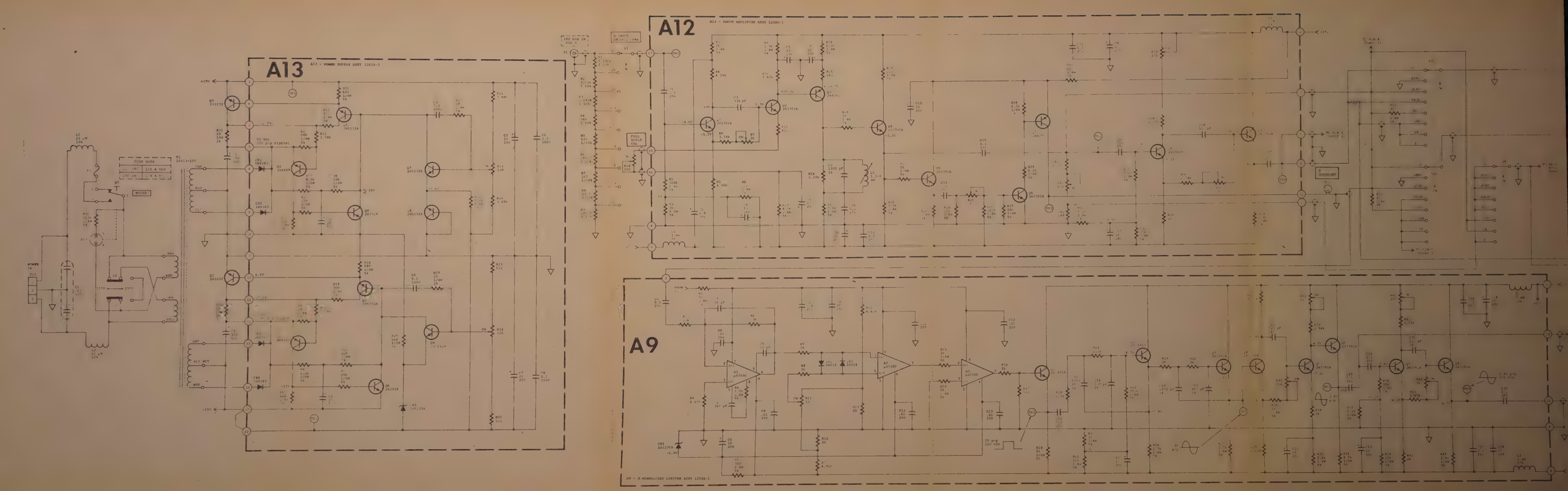
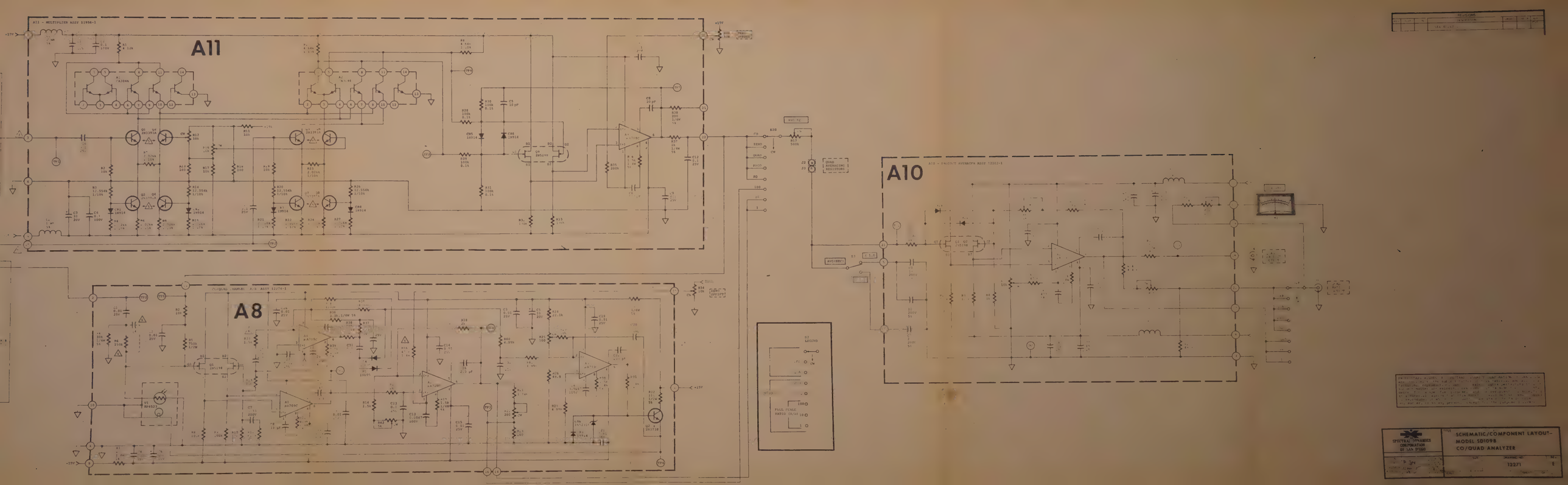
SPECTRAL DYNAMICS CORPORATION OF SAN DIEGO	SCHEMATIC/COMPONENT LAYOUT - MODEL SD109B CO/QUAD ANALYZER
	SIZE D — DRAWING NO 12271 — REV E


A13
A12
A9


A11
A8
A10
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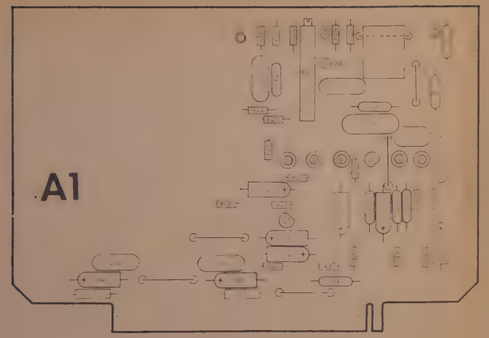
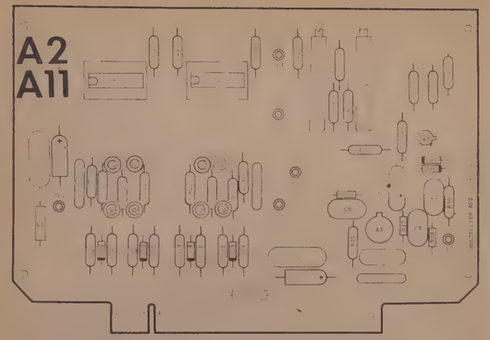
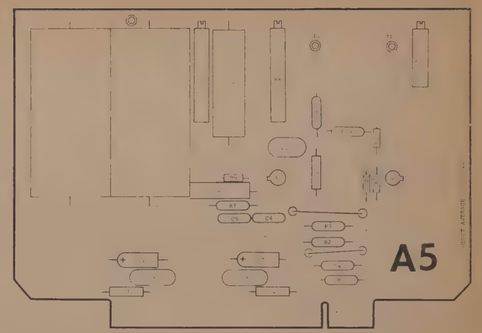
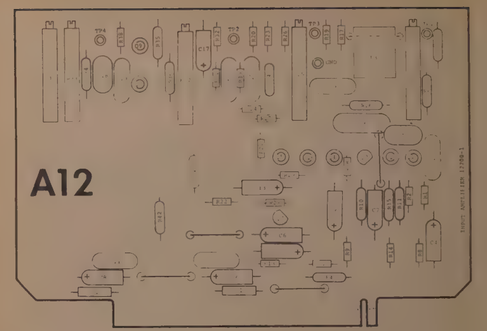
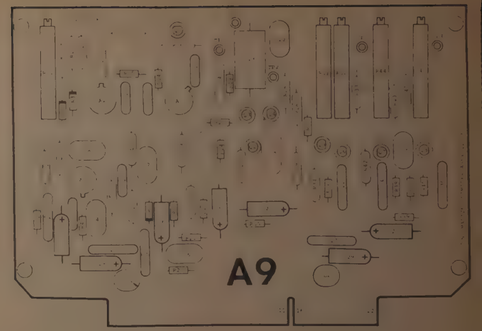
MODEL SD1098
CO/QUAD ANALYZER
SPECTRAL DYNAMICS CORPORATION OF SAN DIEGO
12271

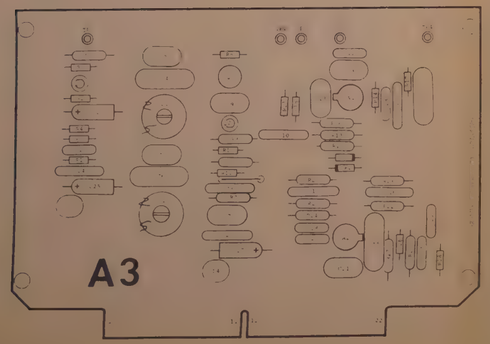


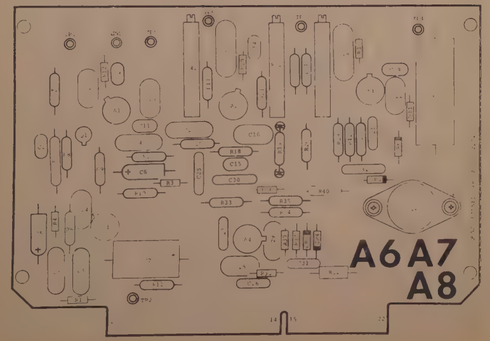


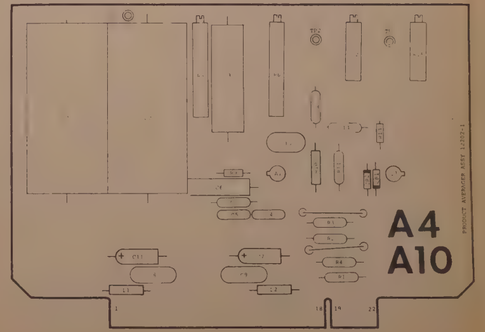


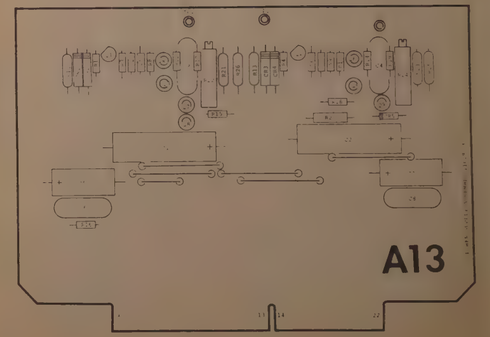












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CONSIDERATIONS IN THE ANALYSIS OF ARBITRARY WAVEFORMS

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By: A. C. Keller, Spectral Dynamics Corporation of San Diego

A. C. Keller

Mr. Keller received the B. S. E. E. Degree from Manhattan College in New York City. He was employed at the White Sands Missile Range for seven years during which time he received the M. S. E. E. Degree from New Mexico State University. Much of his graduate studies were related to spectrum analysis, measurement of probability function and other signal processing techniques. Prior to joining Spectral Dynamics Corporation in November 1965, Mr. Keller was group leader in charge of analysis of all environmental data gathered at the White Sands Missile Range.

INTRODUCTION

All signals can be thought of as existing in three dimensions. As shown in Figure 1, a sinewave is in reality an amplitude-time curve existing at some fixed point on the frequency axis. An examination or projection of this sinusoidal time history on the amplitude-frequency plane would be a vertical line at the frequency f_1. This is the true spectrum of a sinewave and can be viewed as the two-dimensional projection of a time function onto the amplitude-frequency plane. Such a projection of the frequency parameters of a time-amplitude waveform is called a spectrum or frequency analysis.

Similarly the properties of this amplitude-time waveform could be viewed in the amplitude-time domain. One immediate result would be the original sinusoidal waveform. If, instead of viewing the waveform in real time, a pseudo time equal to the time difference between sampled points on the waveform were used, a correlation function would result.

If the arbitrary time function under investigation were viewed in the amplitude-time domain, not with respect to a time difference between points on the function, but with respect to the percentage of time that the function stays within certain amplitude limits, a probability function would result. This again would be a function of amplitude and time and would be truly independent of frequency.

We have, then, three situations which immediately come to light:

In the first case, a signal is investigated for its frequency content. This is done by averaging the signal over a certain time interval and using the appropriate Fourier weighting function. The result of this transformation is a function in the frequency-amplitude domain from which time has essentially been averaged out.

In the second case, an average value is again obtained through integration with respect to time, resulting in a function which is described in the pseudo domain of time difference, tau (τ).

In the third case, the arbitrary time function is operated on in an amplitude and time dependent sense, with a result that is completely independent of frequency but is averaged over time and is in reality plotted on an amplitude-amplitude basis.

Each of the three functions, frequency spectrum, time correlation and amplitude probability, have important engineering significance.

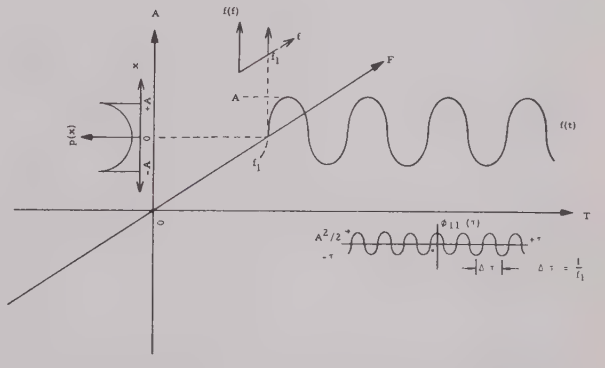

FIGURE 1. THREE-DIMENSIONAL REPRESENTATION OF AN ARBITRARY WAVEFORM.

CORRELATION

Correlation techniques have been used for many years to extract a signal from noise or to reveal the presence of a periodicity in an otherwise completely random signal. Thus, if a signal is correlated with itself (autocorrelation), the random component of the signal will reduce rapidly to zero in the correlation domain, depending on the bandwidth of the noise present, and any periodic component will immediately be revealed as a continuing function in the time delay domain. This can be shown for non-periodic signals as an autocorrelation function $\phi_{11}(\tau)$:

$$(1) \qquad \phi_{11}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t)\, f_1(t+\tau)\, dt$$

Similarly, if two signals are crosscorrelated, one with another, an immediate result is the indication of any and all mutual properties which these signals may share. Thus, a crosscorrelation function $\phi_{12}(\tau)$ is defined

$$(2) \qquad \phi_{12}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t)\, f_2(t+\tau)\, dt$$

where T = time over which observation and integration are performed. One important application of crosscorrelation techniques is the determination of which of many inputs are related to a specific output. If a force is measured at one point on a rocket vehicle and it is necessary to determine which, if any, possible input most directly caused the resultant measured force, a crosscorrelation analysis between the resultant force and each of the possible inputs will reveal which input is most closely related to the measured output force.

PROBABILITY

Probability functions have been extremely useful for many years in the prediction of the behavior and structural response of many tactical vehicles. For example, in the early days of random vibration testing, although it had been shown that the output of random noise generators was truly Gaussian, very little work had been done on the actual estimation or measurement of the probability function associated with in-flight random vibration. Required stress levels were often simulated in the laboratory, based on only a knowledge of the overall rms vibration and the assumption that the probability density function which the data followed was truly Gaussian. In the late 1950's, economic, automatic probability density analyzers first became available. When these analyzers were applied to the analysis of in-flight vibration data, surprisingly perhaps, much of the data was found to truly approach the Gaussian distribution. However, other data which had been readily assumed to follow a Gaussian distribution was seen to be in gross error.

By the same token, many of the tests for equivalence between sinusoidal and random vibration qualification testing have depended to a large extent on the assumption that the random vibration environment seen by a specimen during actual usage indeed followed a Gaussian density function. Once again, this was in many cases a valid assumption; however, the availability now of automatic probability density analyzers and event counters enable the design of qualification and equivalent tests to be approached from a more scientific standpoint.

A probability density function can be defined as:

$$(3) \qquad p(y) = \lim_{\substack{T \to \infty \\ \Delta y \to o}} \frac{1}{T(\Delta y)} \sum_{i=1}^{\infty} t_i \quad (y,\ y + \Delta y)$$

Where t_i is the amount of time the signal stays in an amplitude window Δy units wide.

SPECTRUM ANALYSIS - WHICH FUNCTION?

Although both probability and correlation functions are seen to be extremely useful engineering tools, by far the most widely used and generally understood signal presentation has been a plot of spectral content. Several factors have contributed substantially to this; among these have been the long history of Fourier analysis, the ready availability of wave analyzers for many years, the acceptable signal-to-noise enhancement possible with commercially available analyzers and the close relationship between signal spectrum content and general test specifications. (An additional factor has historically been the difficulty of physically making accurate correlation measurements over wide ranges of amplitude and time delay.)

Once it has been decided that the properties of an arbitrary signal are best described for a specific case through the use of its spectral content, the first question which arises is exactly which amplitude parameter in the frequency domain shall be plotted. If the possibilities are examined, there results a choice between peak values, average values, rectified average values, effective root mean square values or higher order (power) values.

Since acceleration waveforms are often recorded, viewed and compared based on the output of a crystal accelerometer, which is a first power transducer, a first power parameter would seem most amenable to any comparison. It is obviously impractical to speak of the comparison of peak values, since for a random acceleration waveform, the peak acceleration in theory can approach infinity. Likewise, the average acceleration of most stable vehicles is, in most cases, zero. Thus, a choice boils down to that between rectified average values and true root mean square or effective values.

If the probabilities and correlation functions mentioned earlier are examined in a side-by-side comparison with a spectrum function, it is seen that the standard deviation associated with a probability function and the correlation coefficient associated with zero time delay are both related to the effective or true root mean square value of the waveform under analysis. This is shown in Figure 2. In addition, many voltmeters are calibrated to read the true rms value of sinusoidal calibration signals and for years a root mean square value has been the term accepted by hosts of electrical engineers. For these reasons in addition to the close correlation which then results from spectrum, probability and

correlation analysis, the use of a spectrum parameter from which rms values may be determined has long been accepted in the vibration field.

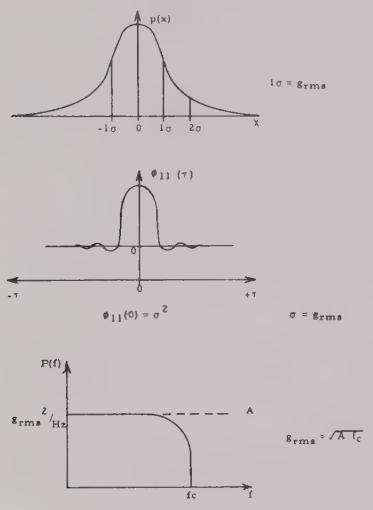

FIGURE 2. EXAMPLE
ANALYSIS OF FILTERED RANDOM ACCELERATION WAVEFORM

Once it has been determined that a final answer in terms of rms acceleration must be achieved a hard look must next be taken at the techniques to be employed in performing a spectrum analysis. Under most conditions a single reading of total wide band root mean square acceleration is not sufficient. If this were sufficient a simple analysis could be quickly made using only a broadband rms meter. However, information must be obtained which relates both to the spectral content of the waveform under analysis and to its effective root mean square value over an arbitrary portion of the total spectrum. Thus, if the root mean square acceleration between two frequencies f_1 and f_2 is available to a design engineer, he has a ready means of comparison between resultant data, test specifications and additional types of analyses such as correlation or probability density which may have been performed.

If one were to examine a spectrum analysis presented in terms of root mean square acceleration versus frequency, an immediate problem would come to light. Although it would be quite straightforward to read off an rms value of acceleration at a specific frequency the determination of root mean square acceleration over a frequency band would be extremely difficult. This is in essence so since voltages and accelerations may not be combined, in general, as a linear addition, but rather must be combined as the square root of the sum of the square of individual components.

As a general rule the only voltages which may be added directly are dc voltages or those vector quantities which are exactly in phase. Since neither of these represents a practical acceleration waveform a frequency spectrum performed on the basis of g_{rms} vs. frequency would not seem to be the answer. Rather a term proportional to a squared voltage, or power, would be more convenient if our intermediate goal is the addition of values over a certain portion of the spectrum. Thus, use of a power term will facilitate the addition of spectrum values between different arbitrary frequencies since direct addition or summation of values is then valid.

However, one additional point remains to be satisfied. This is, that if spectrum values are to be summed between two specific frequencies to achieve a resultant overall level, the final answer depends critically upon the absolute magnitude of the quantities being added. It is known that for a random input the output of a bandpass filter increases with filter bandwidth; thus, if two spectrum analyses were performed on a common data sample each with different bandwidths, the analysis performed with the wider bandwidth would result in a higher apparent overall rms acceleration. This would obviously not be a true result. Naturally it is often desirable to perform analyses with different bandwidths since finer resolution may be obtained by using a narrower filter. However, the process of providing better resolution should be compensated in the analysis to the extent that it does not at the same time provide an erroneous answer. Since for a white noise input the power output of a bandpass filter increases directly proportional to effective bandwidth (thus, the voltage output of a bandpass filter increases proportional to the square root of bandwidth increase) an obvious solution is to eliminate the effect of filter bandwidth on the analysis performed. This is most easily accomplished by dividing the total squared analysis by the effective squared bandwidth used.

We have thus seen that in order to provide a useful quantity which can be related both to other analyses and to test requirements and at the same time arrive at our required data in a simple straightforward manner from a graphical analysis, yet make the analysis amenable to comparison with analyses performed using different filters, a graphical presentation in terms of mean squared acceleration divided by filter bandwidth is in one sense an optimum choice. This term is usually referred to as Power Spectral Density (g_{rms}^2/Hz).

DEFINITION OF PSD

The Power Spectral Density of a signal is independend of the phase of the signal with respect to some arbitrary reference. Thus two signals identical in nature but differing only in relative phase will have identical power spectra. Mathematically, Power Spectral Density is related to the Fourier transform of a signal. Thus, if we define a Fourier transform, F(ω), such that

$$(4) \qquad F(\omega) = 1/2\,\pi \int_{-T/2}^{T/2} f(t)\epsilon^{-j\omega t}\, dt$$

then a power spectral density function can be arrived at as a normalized product between a Fourier transform and a complex conjugate transform as shown in equation 5.

$$(5)\quad P(\omega) = \lim_{T\to\infty} 1/T\,[\,F(\omega)\;.\;F(-\omega)\,]$$

A more satisfying derivation of Power Spectral Density (Ref. 1) is available based on Parseval's theorem. Parseval's theorem, in general, states that the total energy associated with a waveform in the time domain is equal to the energy of the same waveform when translated to the frequency domain. Thus, the average power for a periodic function can be stated as:

$$(6)\quad P_{ave} = \frac{1}{2T}\int_{-T}^{T} [f(t)]^2\,dt$$

For a non-periodic function, $f_n(t)$, this becomes

$$(7)\quad P_{ave} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [f_n(t)]^2\,dt$$

where $f_n(t)$ should be continuous such that

$$\int_{-T}^{T} [f_n(t)]^2\,dt \text{ is finite}$$

and

$$\int_{-\infty}^{\infty} [f_n(t)]^2\,dt \text{ exists.}$$

By Parseval's theorem,

$$(8)\quad \int_{-\infty}^{\infty} [f(\beta)]^2\,d\beta = \int_{-\infty}^{\infty} [F(f)]^2\,df$$

where

$$(8a)\quad F(f) = \int_{-\infty}^{\infty} f(\beta)\,\epsilon^{-j2\pi f\beta}\,d\beta$$

and

$$(8b)\quad f(\beta) = \int_{-\infty}^{\infty} F(f)\,\epsilon^{j2\pi\beta f}\,df$$

are a Fourier transform pair.

Through a change of limits in equation 7 we can write

$$(9a)\quad P_{ave} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [f(t)]^2\,dt$$

$$(9b)\quad = \lim_{T\to\infty} \frac{1}{2T}\int_{-\infty}^{\infty} [f(t)]^2\,dt$$

substituting (8) into (9)

$$(10a)\quad P_{ave} = \lim_{T\to\infty} \frac{1}{2T}\int_{-\infty}^{\infty} [f(t)]^2\,dt$$

$$(10b)\quad = \lim_{T\to\infty} \frac{1}{2T}\int_{-\infty}^{\infty} [F(f)]^2\,df$$

$[F(f)]^2$ is an even function, or in other words the power in the negative side of the two sided spectrum is equal to the power in the positive half. Thus,

$$(11a)\quad P_{ave} = \lim_{T\to\infty} \frac{1}{2T}\,2\int_{0}^{\infty} [F(f)]^2\,df$$

$$(11b)\quad = \lim_{T\to\infty} \int_{0}^{\infty} \frac{[F(f)]^2}{T}\,df$$

$$(11c)\quad = \lim_{\substack{T\to\infty\\ \Delta f\to 0}} \frac{1}{T}\left[F_0(f)^2 + F_1(f)^2 + \ldots F_\infty(f)^2\right]_{\Delta f}$$

$$(11d)\quad = \lim_{\substack{T\to\infty\\ \Delta f\to 0}} \sum_{n=0}^{\infty} \frac{[Fn(f)]^2}{T}\,\Delta f$$

The Power Spectral Density, P(f), is then

$$(12)\quad P(f) = \frac{P_{ave}}{\Delta f}$$

where Δf is the bandwidth of a realizable filter which approaches zero Hz in the limit.

PSD ANALYZERS

In commercial PSD analyzers three steps are performed. First, the signal to be analyzed is introduced to a narrow band filter which is to be swept over the frequency range of interest. Secondly, after filtering, a power function is formed from the filtered signal output. Finally, a normalized average product is formed through integration and division by effective filter bandwidth.

From the preceding derivation, we see that a function P(f) has been defined such that

$$(13)\quad P(f) = \lim_{\substack{T\to\infty\\ B\to 0}} \frac{1}{T}\int_{0}^{T} \frac{y_B^2}{B}\,dt$$

where

$P(f)$ = Power Spectral Density (PSD)

y_B^2 = instantaneous square of signal within a narrow bandwidth, B

B = signal bandwidth (Hz)

This says that the true power spectral density function is obtained by averaging the squared value of a signal in a narrow bandwidth B, over an infinite time period with an infinitesimally small filter bandwidth. In practice, we don't have infinitely long random records nor the time to average to infinity, and we don't have infinitesimally narrow bandwidths. Thus, we can never attain the true PSD.

We must simply estimate the power spectral density function from a finite length of record, averaging over a finite time interval with a finite bandwidth. Equation 13 then becomes

$$(14) \quad \tilde{P}(f) = \frac{1}{T}\int_0^T \frac{y_B^2}{B}\,dt = \frac{\overline{y_B^2}}{B}$$

where

$\tilde{P}(f)$ = an estimate of the power spectral density

T = a finite averaging time

B = the effective noise bandwidth of the analysis filter

$\overline{y_B^2}$ = the average of the square of the amplitudes within the analysis bandwidth

Note that equation 14 assumes a flat spectrum within the passband of the analyzer.

Once a suitable filtering technique has been selected (such as crystal filter/heterodyned, active low-pass filter/modulation etc.), the next consideration is that of performing a power function operation. From the rigorous definition of PSD, a squaring operation would seem to be in order. However, the limits of the power spectral density derived from much of the data available today indicates variations in PSD levels of as much as 80 dB. A true squaring circuit would thus be required to operate over approximately 80 dB of dynamic range. Until very recently, this was completely impossible and even at this time the design of a circuit to operate in a useful manner over this dynamic range is quite impractical. Noise level, linearity and instability are three considerations working against the practical use of wide dynamic range, multiplication circuits.

Let us now investigate some of the properties of filtered random signals to see if these properties can in some way assist us in our analysis.

EFFECT OF FILTERING ON RANDOM SIGNALS

If a broad band random signal which contains no true periodic components is analyzed, several properties become immediately apparent. The spectrum function of the broad band signal may be flat or may have some arbitrary peaks and notches in it. The probability function of the broad band signal would most likely be some arbitrary function, not necessarily Gaussian. By the same token, if the signal were truly random, its autocorrelation function would decrease rapidly to zero for both positive and negative time delay. The application of this signal to a true rms meter would result in a steady indication of the signal rms level. If an oscillograph of the wideband signal were made or if an oscilloscope presentation were carefully analyzed, it would be noted that on the positive side of the signal's mean value, both positive and negative, going peaks occurred and likewise on the negative side of the signal's mean value, positive and negative going peaks would appear.

If the broad band signal is now processed through successively narrower band pass filters several interesting changes occur. In the first place, as the filter bandwidth becomes narrower, the rms meter indication begins to show more fluctuations. Secondly, the spectral content of the filtered signal naturally begins to compress in the frequency domain such that only frequencies within the filtered data bandwidth display significant energy. Thirdly, as the bandwidth of the filter reaches a certain critical width, the oscilloscope or oscillograph presentation no longer indicates negative going peaks on the positive side of the mean value of the signal and vice versa. The signal will in reality begin to appear as an amplitude modulated sine wave with a slight phase deviation. In the fourth place, the autocorrelation function of the filtered signal begins to spread out in the time delay domain. The time over which the signal remains essentially correlated is seen to be an exact function of the bandwidth of the filter. Finally and most important perhaps, it will be noted that as the bandwidth decreases beyond a certain point, the probability density function of the instantaneous amplitudes begin to almost perfectly follow the Gaussian density function.

It has been shown by many test laboratories that if a probability density analysis is performed, for example, on the output of an FM reproduce amplifier when a tape machine has been stopped, the resulting probability function will be in most cases quite distorted and not at all Gaussian. If, however, the noise signal out of the reproduce amplifier is filtered by a bandpass filter placed in the spectrum of the signal where significant energy exists, the probability function of the filter output will approach an almost perfect Gaussian density function. This can be of extremely important significance if one is attempting to measure the effective value of an arbitrary waveform.

If a signal of unknown statistical distribution is examined, the ratio of its rectified average to effective or root mean square value can be almost any quantity. For a sine wave, the ratio of effective to rectified average values is known to be 1.11. Similarly, for a random signal which follows a Gaussian density function it is known that the ratio of rms to average is 1.25. (See Appendix) Thus, average detecting meters, for instance, which are calibrated for use with sine waves and which give erroneous indications upon the application of a wave form other than sinusoidal can be used quite readily to measure effective values if it is known that the input signal follows the Gaussian distribution.

This information then can be used to considerable advantage by an engineer desiring to perform power spectral density estimates on random waveforms through the use of narrow band filters. Since it has

been shown that the output of a narrow bandpass filter when a wide band random signal is applied does truly approach a Gaussian density function, a linear detector with a simple correction factor applied can readily be used in lieu of the squaring or square law detector for the true estimation of power properties of the filtered signal. Thus, for example, if the output of a narrow bandpass filter is detected using an average detector and a correction factor of 1.25 is applied, a resultant quantity will be obtained which is directly proportional to the effective or root mean square value of the filtered signal output. In order to complete the PSD analysis in this fashion, analysis filter bandwidth must be removed from data consideration and a signal presented as a power function for plotting.

Since in many practical cases investigations of structural resonances involve PSD variations of greater than 40 dB, data has been most often presented on a logarithmic (ordinate) basis. This tends to make frequent rescaling unnecessary and provides equal resolution over an entire range of amplitude values.

We have seen then that through a simple calibration technique a linear detector may be applied to the output of a narrow bandpass filter to give a resultant indication in terms of true rms voltage. Also, from equation 12, it is seen that power spectral density is in essence average power divided by effective filter bandwidth. Since the square root of average power is by definition true rms voltage, the squaring of an effective (rms) value followed by bandwidth normalization will indeed result in a true power density function.

Since logarithms are to be employed in a resultant graphical display, the function under analysis can be examined in terms of logarithms.

(15) $\log g^2/\text{Hz} = 2 \log g - \log \text{Hz}$

If g_{rms} is determined and its logarithm taken, doubling this logarithmic value and subtracting the log of the filter bandwidth will present an averaged value of power spectral density in a logarithmic form. Note that if an average detector with a dynamic range of 40 dB is used (as in Spectral Dynamics Model SD101A Dynamic Analyzer), the action of taking log values and doubling the result gives a true indication of power over four full decades of power. If constant bandwidth filters are employed, the log of filter bandwidth represents a known constant which can be readily subtracted during calibration of an X-Y recorder.

In the analysis of structural response it is seen that in many cases the resonant bandwidths excited at different points on a structure remain essentially constant on a logarithmic scale or in other words are of a constant Q variety. Since it is normally sufficient to resolve a resonant peak in a spectrum with a filter bandwidth which is approximately 1/4 the width of the resonant peak, it is conceivably useless or redundant to provide excessive resolution at higher frequencies. For this reason, it has long been felt that the ideal spectrum analyzer for use in many investigations of structural properties would be an analyzer whose selectivity or effective filter bandwidth automatically increased with ascending frequency forming a constant Q or constant percentage analysis. If an analyzer were available whose resolution remained essentially constant on a logarithmic scale (a constant Q analyzer) a method of data averaging would have to be selected for use with the filtering function. Two logical choices would exist. In one case, a constant averaging time could be used while on the other hand an averaging time which decreased with increasing filter bandwidth would be most appropriate. This can be more readily seen if we examine some of the statistical considerations involved in the selection of averaging time and analysis rate.

DEGREES OF FREEDOM

In analysis of a periodic waveform, harmonic amplitudes can be considered to exist at some fixed level for an infinite time length. Thus, the averaging of harmonic amplitudes does not become a consideration since the average of a periodic function is the same whether it is averaged over one or many integer periods. However, when performing an analysis of random waveforms, it must be kept in mind that since the data to be processed is essentially random in nature no exact answer to the frequency properties of the signal may really be determined. Rather an estimate of the spectral content of a signal must be achieved. Since the properties of a random signal are governed by statistical considerations the quality of a power spectral density estimate is likewise governed by many statistical properties. One of the primary and most often misused considerations is that of statistical degrees of freedom (n) associated with a data sample.

In one sense the degrees of freedom associated with a section of data are intimately related to the number of samples required to reconstruct the data signal under analysis. Thus, if a d-c signal were to be analyzed, one sample would be sufficient to predict the behavior of the signal for all eternity. By intuition we could say that this data would have very little "freedom". One of the first to extend this reasoning to arbitrary data signals was H. Nyquist of Bell Telephone Laboratories. He showed that the number of samples required to completely define an arbitrary signal was directly related to the bandwidth of the signal. For a data bandwidth B Hz wide he was able to show that 2 B samples were sufficient to allow the signal to be reconstructed in a time domain sense. Thus, if a signal B Hz wide in frequency and existing for T seconds were to be sampled and reconstructed without ambiguity, a total number of samples equal to 2 BT would be required.

This same reasoning follows over directly into the statistical estimation of the properties of random signals. Here we define statistical degrees of freedom

(16) $$n = 2BT$$

where B is the bandwidth of the data signal in Hz and T is the length of the data sample in seconds. It is intuitively obvious that if we restrict the bandwidth of the signal under analysis or reduce the total time over which we may observe the signal, we are in one sense reducing the freedom of the signal. As the freedom of a random signal is reduced, similarly our ability to accurately specify a property of the random signal is also reduced.

As was mentioned previously, it has been shown in many references that the output of a narrow bandpass filter, when random noise is applied to the input, does truly approach a Gaussian distribution. Since power spectral density measurements will be made employing narrow band filtering, we may then say that the immediate output of our analyzing filter will follow a Gaussian density function. Since we are interested in the square of the signal we are then concerned with the properties of the square of a Gaussian random variable. If a random variable exists which follows a Gaussian density function, it has been shown that the square of this variable follows what is known as a Chi-square (χ^2) distribution. As the freedom, or statistical degrees of freedom, associated with our data samples varies, our ability to accurately specify a function of the squared value of the signal we are analyzing is directly related to the Chi-square distribution. This is shown more fully in Figure 3. Note that for a given statistical confidence, our ability to accurately specify the true power spectral density of a signal increases in direct proportion with the statistical degrees of freedom of our signal.

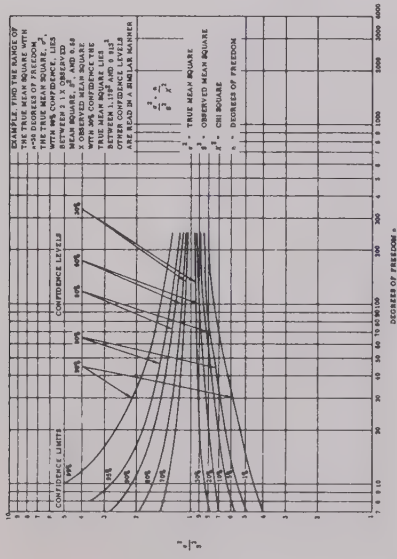

FIGURE 3.

CHI-SQUARE DISTRIBUTION CHART

AVERAGING

Knowing that the statistical freedom of our resultant analysis is based on filter bandwidth and data length, two of the first considerations which must be answered is what length of averaging shall be used and how fast may an analysis be performed. Essentially two types of averaging are available for use in performing PSD estimates. These are true integration over a fixed integration time and continuous RC smoothing or averaging. By using true integration, a point on the PSD estimate is achieved following each integration time. On the other hand, RC averaging uses the weighted results of all signal amplitudes that have gone before. Thus, true integration will result in one plot point after each integration time, whereas continuous or RC averaging will produce a continuous spectral presentation which in many instances is easier to interpret.

It has been shown in Reference 2 (Section 6) that in order to produce an unbiased estimate of a mean squared value, statistically equivalent variances will be achieved if an effective integration time, T, equal to twice the RC circuit time constant is used. This result was shown to be independent of bandwidth B and thus gives an equivalent expression for statistical degrees of freedom as

(17) $$n = 2BT = 4BRC$$

Thus, a time constant RC = T/2 can be used. A larger time constant will not improve the statistical quality of the measurement whereas a shorter time constant will definitely reduce the statistical quality of the measurement.

COMPARISON OF ANALYZERS

A. Fixed Bandwidth, Fixed Averaging Time

If a data sample T seconds long were to be analyzed using a bandwidth B cycles wide, intuition alone would indicate that it would be desirable for the entire data sample to pass through the filter bandwidth prior to averaging. Thus, using true integration we would arrive at an analysis or sweep rate of one filter bandwidth per data sample. That is,

(18) $$AR = B/T$$

If RC averaging is used it must be kept in mind that it takes approximately 4 time constants (4 RC seconds) for the output of an RC averaging circuit to build up to 98% of its final output. The maximum analysis rate (sweep rate) for the RC averaging case then becomes

(19) $$AR \leq B/4RC$$

Note that since we have selected RC = T/2, this result states that when using true integration a sweep rate twice as fast as that used during RC integration is possible. These sweep rates, of course, apply to the use of a

constant bandwidth filter with resultant linear sweep rate. The total analysis time (A_t) in this case would then be

(20) $$A_t = \frac{1}{AR} \left[f_2 - f_1 \right]$$

where AR is analysis rate. Using a fixed bandwidth and constant RC averaging time the statistical degrees of freedom would then remain constant over the entire range of analysis.

B. Constant Percentage Bandwidth, Fixed Averaging Time

The equivalent results obtained when using a constant percentage bandwidth analyzer with fixed averaging time have been derived in Reference 3. If p is the percentage of bandwidth with respect to frequency of operation (f_0), the corresponding equations become

(21) $$n = 4pf_0RC$$

(22) $$AR \leq pf_0/T \; ; \; AR \leq pf_0/4RC$$

(23) $$A_t = \frac{T}{p} \ln \left[f_2/f_1 \right] = \frac{4RC}{p} \ln \left[f_2/f_1 \right]$$

This shows that the statistical confidence level increases directly with the center frequency of the analyzer. It also points out that the sweep rate is a first order function of the analysis frequency or is in fact a logarithmic sweep. Thus, we have set up a condition where filter bandwidth and analysis rate are continuously increasing but at the same time statistical confidence is also increasing. If we wish to provide a constant percentage analysis with constant statistical accuracy, a solution would be to provide decreasing RC averaging time as frequency is increased.

C. Constant Percentage Bandwidth, Tracking Averaging Time

This situation has also been examined in Reference 3 and results in a constant percentage type analysis in which bandwidth is continuously increased, sweep rate is continuously increased and averaging time constant is continuously decreased. Although, at the present time, this type of analyzer operating over wide frequency and dynamic range is beyond the state of the art, it is interesting to note the results which are obtained from analysis of such an operation. If k is defined as a constant depending upon the desired statistical confidence level, f_0 is the analyzer filter center frequency and p is the percent bandwidth with respect to f_0, then the corresponding equations for statistical degrees of freedom, analysis rate and total analysis time become

(24) $$n = 4kp$$

(25) $$AR = pf_0^2/4k$$

and

(26) $$A_t = \frac{4k}{p} \left[\frac{1}{f_1} - \frac{1}{f_2} \right]$$

where

(27) $$k = f_0RC$$

D. Programmed Fixed Bandwidth/Variable Averaging Time

There are naturally cases where a constant percentage resolution analyzer is not desirable. This would be the case for example when performing an analysis of periodic waveforms. If a continuously increasing bandwidth filter were used, higher harmonics would tend to become intermingled within the filter bandwidth thus giving erroneous results. A completely versatile analysis system would then of necessity contain both fixed bandwidth filters and programmably increasing bandwidth filters.

From a practical standpoint, such a system has been developed and is the Spectral Dynamics Corporation Model SD1001-2 Automatic Power Spectral Density Analysis System (Figure 4). This system can automatically switch between five preselected constant bandwidth analysis filters. Sweep rate, averaging time constant and bandwidth normalizing division are also automatically programmed, in addition to performing bandswitching, in the SD1001-2 system.

The analysis equations for each of the programmed filters within the SD1001-2 system are the same as for the fixed bandwidth, fixed averaging time situation case A. Thus,

(28) $$RC = \frac{n}{4B}$$

(19) $$AR \leq \frac{B}{4RC}$$

(20) $$A_t = \frac{1}{AR} \left[f_2 - f_1 \right]$$

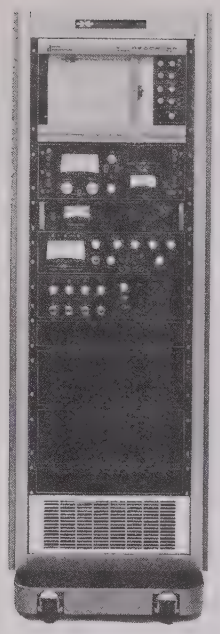

FIGURE 4
SD1001-2

ANALYSIS COMPARISON

As an example, let us examine a typical analysis program as performed on the four types of analysis systems discussed. It is assumed that a random acceleration data sample has been recorded on a tape loop of length Tr equal to 10 seconds. We wish to analyze this from 10 Hz to 2,000 Hz with a desired number of degrees of freedom of 100. A maximum structural Q of 5 is expected which in turn will serve to place some limits on the resolution required in the constant Q type analyzers. Since an analysis bandwidth equal to one-fourth the bandwidth of the narrowest resonance shown in the spectrum should be used, an analysis Q, (QA), of 20 is required. This requires a percent resolution of 5%. At the low frequency end of the analysis (10 Hz) this would require an analysis bandwidth of 0.5 Hz.

A. In the first situation, an analysis will be performed using one constant bandwidth filter and no band switching. If a 5 Hz filter is used, minimum resolution is not attained until 100 Hz.

(28) $$Q_A = 20 = \frac{f_0}{B} \; ; \; f_0 = 20B = 100 \text{ Hz}$$

Naturally above 100 Hz, resolution far better than that required will be the result. With the 5 Hz filter we then calculate an averaging time of 5 seconds ($RC = \frac{n}{4B}$) and a sweep rate of 1/4 Hz per second ($AR = \frac{B}{4RC}$). This results in a total analysis time of 7960 seconds ($A_t = \frac{1}{1/4}\,[2000 - 10]$) or almost 133 minutes. This is an extremely impractical result but unfortunately the only type of analysis provided on many older type analyzers.

B. Let us next examine the same analysis if it is performed using a constant percentage analyzer with fixed RC averaging. The statistical degrees of freedom, n, are once again

(29) $$n = 4pf_0RC$$

Since a 5% analysis bandwidth has been specified, let us assume that the minimum desired statistical degrees of freedom n = 100 is first achieved at 30 Hz. The necessary RC time constant then becomes

(30) $$RC = \frac{100}{4(0.05)(30)} \cong 16 \text{ seconds}$$

The total analysis time

(31) $$A_t = \frac{4RC}{p} \ln \left[f_2/f_1 \right] = \frac{4(16)}{0.05} \ln 200$$

is then 6784 seconds or 113 minutes. This is a slight improvement over the fixed bandwidth situation, although in this case an optimum analysis resolution has been achieved throughout almost the entire display.

C. In the next case, let us examine the constant percentage bandwidth analyzer using tracking RC averaging. As was previously seen, in this case

(32) $$RC = k/f_0$$

where k is a constant depending upon the degree of statistical confidence desired. Then the total degrees of freedom are

(33) $$n = 4pk$$

Again selecting 100 degrees of freedom and a 5% analysis bandwidth $k = \frac{100}{4(0.05)} = 500$. Our largest averaging time would then again be selected to occur at 30 cycles and would be $RC = k/f_0 = \frac{500}{30}$ or approximately 16 seconds.

Note that at the upper end of the analysis, RC would have reduced to $RC = \frac{R}{f_u} = \frac{500}{2000} = 0.25$ seconds. The total analysis time in this case is then $A_t = \frac{4k}{p}\left[\frac{1}{f_1} - \frac{1}{f_2}\right]$. This becomes almost exactly 4,000 seconds or 66-2/3 minutes. This is approximately half the time required if a single constant bandwidth filter were used for analysis or if a constant percentage analyzer with fixed RC averaging were used.

D. As the final case, we will examine the condition of preprogramming and switching between five fixed bandwidth analysis filters. Assume that available bandwidths are 1.5, 5, 10, 20 and 50

Hz. Although we will not be able to meet all the resolution requirements of a 5% resolution analysis, a realistic compromise can be readily achieved.

The crossover frequencies at which switching between filters should occur is given by

(34) $$f_0 = \frac{B}{p}$$

To meet the minimum resolution requirements using the five available filters, switching should take place at 100, 200, 400 and 1000 Hz. However, in the interest of analysis time and statistical confidence switching will actually be performed at 40, 200, 400 and 1000 Hz. Averaging times are then computed using $RC = \frac{n}{4B}$. We wish to set n = 100; however, for the first filter the maximum "freedom" available is

(35) $$n_{max} = 2BT_r$$

where T_r is the tape loop length.

The entire set of analysis parameters used to control the SD1001-2 system for this test are shown in the following table:

SD1001-2 PROGRAMMED ANALYSIS PARAMETERS

BW	Freq. Range	n	RC	AR	A_t	Analyzer Calib.
1.5	10 Hz - 40 Hz	30	5	0.075	400	0.7
5	40 Hz - 200 Hz	100	5	0.25	640	.384
10	200 Hz - 400 Hz	100	2.5	1	200	.271
20	400 Hz - 1000 Hz	100	1.25	4	150	.192
50	1000 Hz - 2000 Hz	100	0.5	20	50	.121

Total analysis time = 1440 seconds = 24 minutes

The parameters actually set into the SD1001-2 are then BW, RC, AR, Analyzer Cal and Bandswitching frequencies. The statistical accuracy has been kept essentially constant throughout with a programmed linear sweep rate used for each filter. In this example, we first meet the requirement for maximum resolution at 30 Hz; resolution has been somewhat sacrificed up to that point.

A. Single fixed bandwidth filters - 133 minutes

B. Constant Q fixed RC - 113 minutes

C. Constant Q tracking RC - 67 minutes

D. SD1001-2 System - 24 minutes

COINCIDENT AND QUADRATURE SPECTRUM ANALYSIS

Thus far, the main point of discussion has been that of analyzing one signal for its spectrum, time correlation or amplitude content. It was shown in equation 5 that a power spectrum measurement involves the product of a Fourier transform of a signal and its conjugate transform. Thus, any complex or imaginary component of a signal (with respect to the Fourier weighting function) is offset by an equal but opposite component in the conjugate. The result is that a power spectrum estimate is a real, even function. That is, it is independent of phase. Thus, two signals differing only in phase will have identical power spectra.

However, in many practical cases in engineering, information relating two signals is required. Thus, any time the transfer function between two points in a system is required, the measurement of input/output relationships or between-signal properties is required.

In the frequency domain, one measurement of mutual properties between signals is given by the cross-spectrum or cross-spectral density function. In general, most practical circuits and systems (linear or non-linear) are not made up of purely dissipative components and thus have a complex transfer function. The cross-spectrum properties will then likewise, in most cases, contain complex terms or in other words have both real and imaginary components.

From the definition of cross correlation given by equation 2, one definition of cross-spectral density may be stated as

(36) $$P_{12}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{12}(\tau)\, \epsilon^{-j\omega\tau} d\tau$$

The cross-spectral density function $P_{12}(\omega)$ is thus the Fourier transform of the cross-correlation function $[\phi_{12}(\tau)]$. Since $P_{12}(\omega)$ is in general complex, it may also be written either

(37) $$P_{12}(\omega) = C_{12}(\omega) - jQ_{12}(\omega)$$

where $C_{12}(\omega)$ is the co component and $Q_{12}(\omega)$ is the quad component or as

(38) $$P_{12}(\omega) = |P_{12}(\omega)|\, \angle\, \phi(\omega)$$

where

(39a) $$|P_{12}(\omega)| = \left(C_{12}(\omega)^2 + Q_{12}(\omega)^2\right)^{1/2}$$

and

$$(39b) \qquad \phi(\omega) = \tan^{-1} - \frac{Q_{12}(\omega)}{C_{12}(\omega)}$$

Equation 37 gives the cross spectral density function as a CO and QUAD component whereas in equation 38 the cross spectral density is shown as a magnitude and phase component. These are equivalent expressions.

In practice a presentation in terms of CO and QUAD components is often preferred. This is many times the case because of difficulty in detecting and presenting a phase angle near 0° or 360° (as required in equation 39) and because CO and QUAD components may be plotted versus one another (as shown in Figure 5) to form a Nyquist diagram. Nyquist diagrams are extremely useful in predicting the stability behavior of a system.

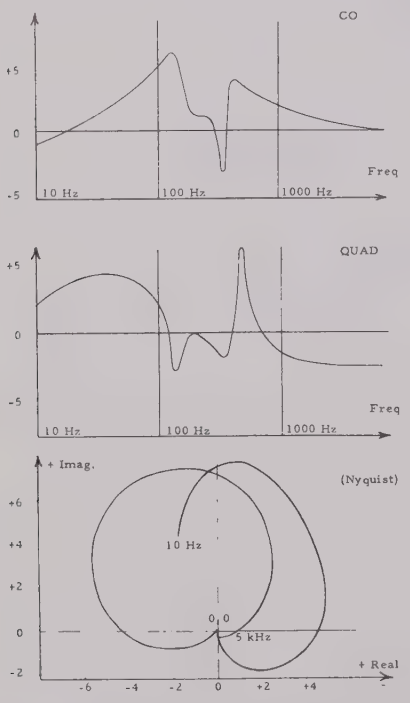

FIGURE 5: CO/QUAD COMPONENT PRESENTATION

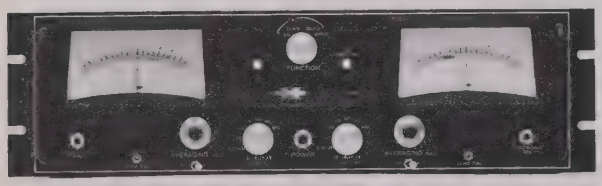

FIGURE 6

SD109

A commercially available analyzer for determining cross spectrum information between 2 signals is the Model SD109 CO/QUAD Analyzer available from Spectral Dynamics Corporation. Designed to operate with two Spectral Dynamics Tracking Filters, the Model SD109 operates on both random and periodic signals and presents two outputs in terms of in phase and quadrature components between signals. The SD109 is shown in Figure 6.

CONCLUSIONS

We have thus seen that although three types of analysis (spectrum, correlation and probability) are quite useful, several reasons dictate the more broad usefulness of spectrum analysis over correlation and probability. Having decided on the performance of a spectrum analysis, it has been shown that in general the most useful term for describing the spectral content of an arbitrary random signal is power spectral density in terms of g_{rms}^2/Hz. Several types of analyzers have then been investigated including constant bandwidth, fixed RC averaging; constant percentage bandwidth with fixed RC averaging; constant percentage bandwidth with tracking RC averaging; and multi-filter fixed bandwidth analyzers with programmed averaging time constant.

The use of constant percentage bandwidth analyzers with tracking RC averaging in one sense represents the ideal analyzer in terms of optimum resolution constant statistical accuracy and minimum analysis time. On the other hand, the use of a programmable fixed bandwidth analyzer using linear detectors in a logarithmic display of g^2/Hz was seen to provide excellent dynamic range and a remarkable saving in total time required for a specific analysis.

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APPENDIX

Relationships Between Average + RMS Values

A. SINEWAVE

For a sinewave of peak amplitude A defined by

$$f(t) = A \sin \omega t$$

an average value, f_{ave}, may be derived as

$$f_{ave} = \frac{1}{T} \int_0^T A \sin \omega t \, dt$$

where T is the period of the sinewave.

Then

$$f_{ave} = \frac{2}{T} \int_0^{T/2} A \sin \omega t \, dt$$

$$= \frac{2}{T} \left[- A \frac{\cos \omega t}{\omega} \right]_0^{T/2}$$

since $\omega = 2\pi f = \frac{2\pi}{T}$ this becomes

$$f_{ave} = \frac{2AT}{T2\pi} \left[- \cos \frac{2\pi}{T} \frac{T}{2} + \cos 0 \right]$$

$$= \frac{A}{\pi} \left[2 \right] = \frac{2A}{\pi} = 0.636A$$

Similarly a root mean square value f_{rms} may be derived as

$$f_{rms} = \left(\frac{1}{T} \int_0^T \left[A \sin \omega t \right]^2 dt \right)^{1/2}$$

$$= \left(\frac{1}{T} \int_0^T \left[A^2 \sin^2 \omega t \right] dt \right)^{1/2}$$

$$= \left(\frac{A^2}{T} \int_0^T \left[\frac{1 - \cos 2 \omega t}{2} \right] dt \right)^{1/2}$$

$$= \left(\frac{A^2}{2T} \left[t - \frac{\sin 2 \omega t}{2 \omega} \right]_0^T \right)^{1/2}$$

$$= \left(\frac{A^2}{2T} \left[T - \frac{\sin 2\pi T}{T} + \sin 0 \right] \right)^{1/2}$$

$$\frac{4\pi}{T}$$

$$= \left(\frac{A^2}{2} \right)^{1/2} = \frac{A}{\sqrt{2}} = 0.707 A$$

Thus the ratio of rms to average value for a sinewave is

$$\frac{0.707A}{0.636A} = 1.11$$

Note that the mean square value is $A^2/2$ and the square of the mean (average) value is $\frac{4A^2}{\pi^2}$.

mean square = $0.5A^2$
square of mean = $0.4053 A^2$

ratio of mean square to square of mean = $1.23 = (1.11)^2$

B. GAUSSIAN RANDOM VARIABLE

The Gaussian density function is described by

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \epsilon^{-(x - \mu)^2/2\sigma^2}$$

where x is the instantaneous amplitude of the random variable under analysis, μ is its mean value and σ^2 will be shown to be the variance from the mean.

That the mean value of the Gaussian distribution is truly μ can be shown by solving for the first moment, m_1. Thus,

$$m_1 = \int_{-\infty}^{+\infty} x p(x) \, dx$$

$$= \int_{-\infty}^{+\infty} \frac{x \epsilon^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \, dx$$

To solve this integral, a change of variables is required such that

$$y = \frac{(x - \mu)}{\sigma\sqrt{2}} \; ; \; dx = \sqrt{2}\sigma dy$$

Then

$$m_1 = \int_{-\infty}^{+\infty} \frac{(\sqrt{2}\,\sigma y + \mu)}{\sqrt{\pi}} \epsilon^{-y^2} \, dy$$

since

$$\int_{-\infty}^{+\infty} y \epsilon^{-y^2} dy \equiv 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \epsilon^{-y^2} dy \equiv \sqrt{\pi}$$

$$m_1 = \frac{\mu}{\sqrt{\pi}} \cdot \sqrt{\pi} = \mu$$ and the mean value is truly μ.

The second central moment or variance, μ_2, is

$$\mu_2 = \int_{-\infty}^{+\infty} \frac{(x - \mu)^2 \epsilon^{-(x - \mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} \, dx$$

using the same change of variables as above

$$\mu_2 = 2 \int_{-\infty}^{+\infty} \frac{\sqrt{2}\sigma^3 y^2}{\sigma\sqrt{2\pi}} \epsilon^{-y^2} \, dy$$

$$= \frac{4\sigma^2}{\sqrt{\pi}} \int_0^{\infty} y^2 \epsilon^{-y^2} \, dy$$

As shown in Reference 5, page 68

$$\int_0^\infty y^2 \epsilon^{-y^2} dy = \frac{\sqrt{\pi}}{4}$$

thus

$$\mu_2 = \frac{4\sigma^2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{4} = \sigma^2$$

The standard deviation (rms value) is then

$$x_{rms} = (\sigma^2 - \mu^2)^{1/2}$$

For a mean value of zero, the rms value is then simply σ.

From the symmetry of the distribution, the rectified average value can be found by

$$x_{ave} = \frac{2}{\sqrt{2\pi}} \int_0^\infty \frac{x\epsilon}{\sigma}^{-x^2/2\sigma^2} dx$$

assuming a mean value, $\mu = 0$

substituting $y = x/\sigma\sqrt{2}$

$$x_{ave} = 2\sqrt{\frac{2}{\pi}}\, \sigma \int_0^\infty y\, \epsilon^{-y^2} dy$$

$$= 2\sigma\sqrt{\frac{2}{\pi}} \left[\frac{\epsilon^{-y^2}}{(-2)}\right]_0^\infty$$

$$= \sqrt{\frac{2}{\pi}}\, \sigma = 0.798\, \sigma = x_{ave}.$$

Thus, the ratio of rms to average value for a random signal following a Gaussian density function is

$$\frac{rms}{avg} = \frac{\sigma}{0.798\, \sigma} = 1.25$$

The ratio of the mean square to the square of the (rectified or absolute) mean is then

$$\sigma^2 = (1.25\, x_{ave})^2$$

$$\sigma^2 = 1.57\, x_{ave}^2$$

Vector Component Techniques: A MODERN WAY TO MEASURE MODES

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sound and vibration

Vector Component Techniques: A MODERN WAY TO MEASURE MODES

Anton C. Keller, Spectral Dynamics Corporation, San Diego, California

The analysis of vector quantities in terms of their real and imaginary components represents a powerful technique to define normal modes, to determine structure damping characteristics, and to provide unambiguous measurements of vector phase angle in complex system analysis. Similar measurements on cross-spectral density analyses of random waveforms can identify the true source of a response through determination of the mutual properties of the forcing and response functions.

IN THE ENGINEERING SENSE, a vector is defined as a measurable quantity having both a magnitude and a direction. Vector quantities quite often measured include acceleration, velocity, and the transfer functions between input and output of a system. In its simplest form, a vector can be broken down into either Cartesian or polar coordinates. For example,

$$A + jB = C \angle \theta$$

where

$$C = \sqrt{A^2 + B^2}$$

and

$$\theta = \tan^{-1} \frac{B}{A}$$

In another sense,

$$A + jB = |C|(\cos \theta + j \sin \theta)$$

Several examples will now be given where the use of vector components (A, jB) has significant advantages over other types of data presentation.

Modal Studies

Modal studies have always been an important part of the design and development stage of large aircraft and space vehicles. Modal patterns are used in flutter analysis, in the computation of dynamic loads, and in laboratory verification of the parameters of an analytic design.

A single, normal mode exhibits the property of having the same phase response relative to the forcing input at all points on the structure. The relationship between response points may be either 0° or 180°. The actual phase relationship at the frequency of modal response between forcing input and the response measurement may be either 0° (for force and velocity) or 90° (for force and acceleration).

A typical normal mode—single-degree-of-freedom response—is shown in Figure 1. The graph, representing the relationship between applied force and resultant velocity, includes the overall magnitude transfer ratio and the phase angle between applied force and resultant velocity. The magnitude ratio peaks at the resonant frequency, and the phase

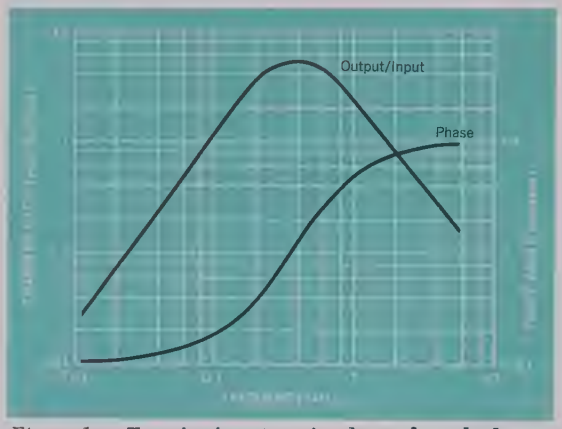

Figure 1 — Transfer function of a damped single-degree-of-freedom mechanical system showing the magnitude ratio and phase angle of the output velocity to input force.

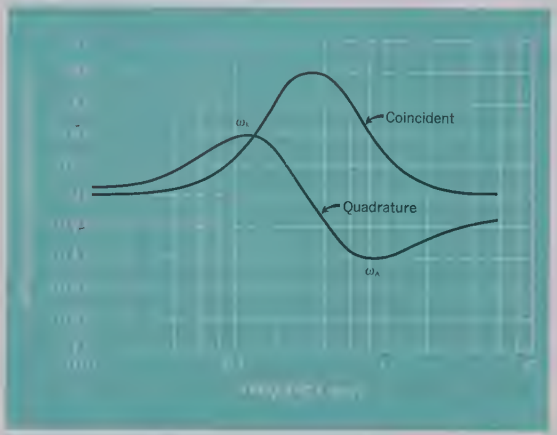

Figure 2 — Normalized transfer product of a single-degree-of-freedom system in terms of the in phase (real) and quadrature (imaginary) components of the transfer ratio vector.

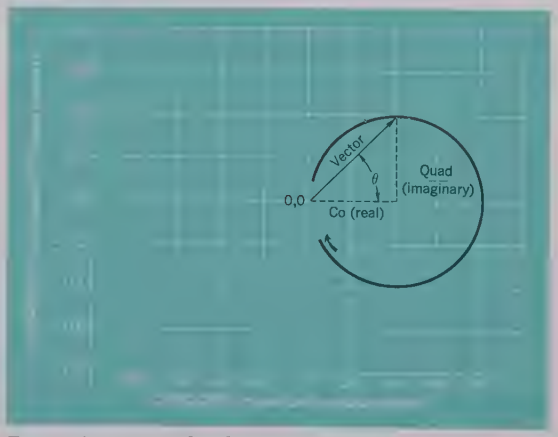

Figure 3 — Normalized transfer product of a single-degree-of-freedom system in terms of polar coordinates (Nyquist diagram).

which began at $-90°$ goes through zero at resonance and eventually attains a $+90°$ relationship.

If we define the force and velocity functions as

$$\text{Force} = F \sin \omega t$$

and

$$\text{Velocity} = V \sin (\omega t + \theta)$$

the graph of Figure 1 describes

$$|\frac{V}{F}| \ (f) \text{ and } \theta(f)$$

The input/output relationship between these two signals may also be defined in terms of the products $FV \cos \theta$ (coincident or in-phase components) and $FV \sin \theta$ (quadrature or 90° component). These are shown in Figure 2. (Remember that with a constant input signal, such as is used during many transfer function measurements, the product, FV, takes the identical form as the quotient, V/F.)

In the case at hand, the coincident (Co) function achieves the peak response at resonance (force and velocity are inphase) and the quadrature (Quad) function goes to zero at the natural frequency of the system. Note that the definition of the Co peak is significantly sharper than that of the overall response shown in Figure 1. This is partly so because the overall response actually represents the square root of the sum of the squares of the Co-squared plus the Quad-squared components.

The real and imaginary components may also be plotted as functions of one another in polar form, as shown in Figure 3. In this case, a normal mode is revealed immediately in the polar domain as a perfect circle. It has been shown[1] quite conclusively that a significant increase in resolution and definition of modal patterns is achieved by using coincident and quadrature components instead of overall magnitude transfer measurements. This is shown in Figure 4. The reason for this is that, even if a frequency-selective system is used during the analysis (such as a tracking filter), the response at a given frequency, when examining bending modes, may quite often be masked by the response at the same frequency caused by adjacent translation and pitch modes. On the other hand, with a display that is both amplitude- and phase-sensitive, such as $AB \cos \theta$ or $AB \sin \theta$, only that response which bears the amplitude and the phase relationships corresponding to simple modal response will be revealed at the frequency of excitation.

The Co and Quad component method actually breaks out the response magnitude being experienced in the direction of interest. Only planar response and 90° motion are preserved—all other confusing components are discarded.

Several means of calculating the damping of a structure are also available directly from a Co and Quad type of presentation.[1] For example, by examining the peak and notch of the quadrature response in Figure 2 and defining the maximum Quad peak as ω_B and the Quad minimum as ω_A, the damping coefficient ratio, ζ, is determined from

$$\zeta = \frac{(\omega_A/\omega_B)^2 - 1}{(\omega_A/\omega_B)^2 + 1}$$

Another graphical technique for determining damping information from a polar display of structural response has been developed and is shown in Figure 5. This technique is based on the observation that, as a structural resonance is excited, the rate of change of phase with respect to frequency between the vectors representing the forcing function and their response motion is a maximum. If we provide external marks on our polar display representing equal increments of frequency, the exact frequency of a mode can be determined as that point along the locus of the vector which has a

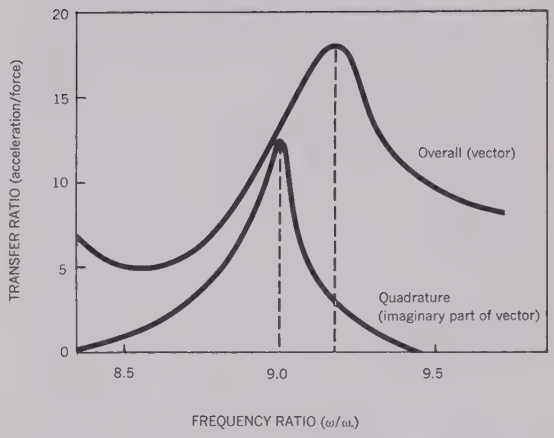

Figure 4 — Comparison of the frequency response curves of a mode in terms of the overall acceleration/force ratio vector and the quadrature component.

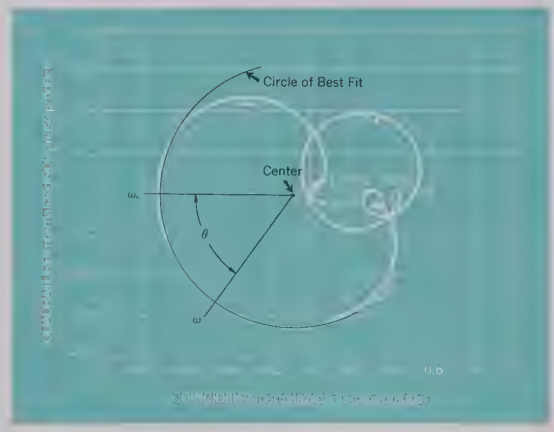

Figure 5 — The damping coefficient, ζ, of a structural resonance can be found from a polar diagram of the transfer function by determining the angle between the vector at the resonance frequency of interest, ω_n, and a vector of known frequency, ω, near the observed resonance frequency.

maximum spacing between frequency markers.

As discussed earlier for a single-degree-of-freedom system, the vector display in polar form will transcribe a perfect circle. In many practical systems where complex modal response is being excited, the polar diagram will transcribe a somewhat distorted circular pattern. In these cases, an excellent estimate of structural damping can be obtained by using four steps:[5]

1. Construct a circle whose circumference is a best fit on the vector curve in the area of resonance (ω_n), and locate the center of this circle.
2. Select a point, ω, of known frequency on one side of the observed resonant frequency, and draw radii from the center of the circle to this point and to the resonant point.
3. Measure the angle, θ, between these two radii.
4. Set up the equation:

$$\text{Damping coefficient ratio} = \zeta = 2c/c_c = \frac{[1 - (\omega_n/\omega)^2]^2}{\tan \theta/2}$$

Additional information concerning simple modes, complex modes, adjacent modes, and various damping factors is given by Kennedy and Pancu.[2]

Unambiguous Measurements of Phase in Complex Systems

In many commercially available phase meters, phase is detected as a function of the zero axis crossings of the input signals. Thus a Schmidt trigger is made to turn on and off as a function of the difference in time between the zero crossings. This means that, as the relative phase between input signals approaches either 0° or 360°, the triggering circuits are made to turn on and off almost simultaneously. Thus for any phase meter that is made to cover a continuous 0° to 360° range, an area of ambiguity exists, at both 0° and 360°. For either a continuous network or a continuous structure such as a cantilever beam, a measurement of amplitude and phase response over a reasonable range of frequencies will include many areas of phase discontinuity.

Since amplitude sensitivity is also an important factor in phase detection,[3] confusing points of discontinuity are also encountered, if one or both input amplitudes go to zero. To interpret accurately the results of a phase measurement, individual amplitudes as well as phase between signals must be monitored. An example of a complex four-pole network transfer function is shown in Figure 6. Note that right at the two points of maximum interest—that is, resonance or peak response conditions (5000 Hz) and nodal response (160 Hz)—the phase meter reaches a point of ambiguity and begins flipping between 0° and 360°. Although in some detecting systems a phase offset may be used to limit the total phase excursion to stay within the useful range of a phase meter, many practical cases exist whereby the phase actually changes over more than 360°, thus automatically causing a point of ambiguity.

The same information is shown in Figure 7, in terms of Co and Quad response. Note that the Co and Quad functions display no discontinuities. This is because, by their very definition, the Co and Quad components defined as $AB \cos \theta$ and $AB \sin \theta$ have to be continuous.

At any frequency, an unambiguous reading of phase may be achieved as the arc tangent of Quad/Co while noting the relative magnitude and polarity of the Co and Quad components.

An even more direct way of describing the actual phase angle under any conditions of continuous phase roll is shown in Figure 8. A polar (or Nyquist) diagram gives a direct reading of the phase in any of the four quadrants. Although frequency is not usually shown directly in a polar diagram, frequency information is available from a simultaneous comparison of Figures 7 and 8.

However, two techniques have recently been developed which permit a continuous display of excitation frequency to be superimposed directly on a polar diagram. The first of these requires the use of a two-pen recorder and is shown in Figure 9. In this type of presentation, one pen of the recorder is used to display Co-versus-Quad data, with the Co signal driving the recorder carriage and the Quad signal driving one of the pens. A dc signal proportional to frequency or to the log of frequency is then used to drive the second recorder pen.

To use this technique, it is necessary only to keep track of the direction of rotation of the vector data. The pen, which is giving a continuous indication of the test frequency, will exhibit monotonic motion in a vertical direction, since frequency is continuously increasing, but will travel from left to right in accordance with the coincident or in-phase vector component, since it is on the same carriage as the Co input. At any frequency, it is necessary only to find the corresponding Co point on the vector plot which will be lined up vertically with a given frequency point. This immediately establishes the frequency associated with any point on the vector dispay. If the vector information shrinks to zero over a portion of the frequency range, this will be indicated by the fact that the frequency marker will traverse a vertical line along the centerline of the graph over this portion of the vector display. In the example shown, the frequency axis, which has calibration markers on the right of the chart, has been displayed in logarithmic form to give good resolution over the entire two and a half decades of frequency sweep. The frequency axis could also be calibrated in linear fashion if only a small range of frequencies is of interest. The obvious advantage of this technique is that it requires no external frequency markings to be placed on the polar data directly.

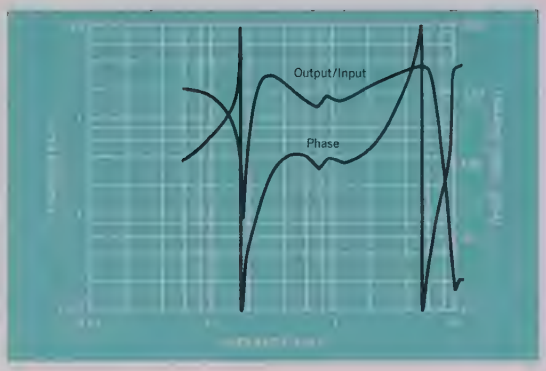

Figure 6 — Transfer function of a complex four-pole network in terms of the magnitude ratio and phase angle of the output to the input.

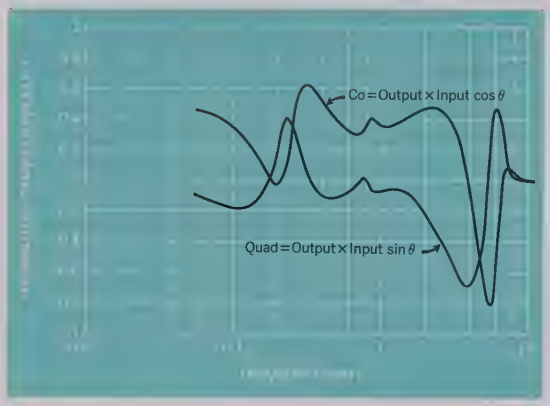

Figure 7 — Normalized transfer product of a four-pole network in terms of the coincident (real) and quadrature (imaginary) components of the transfer ratio vector.

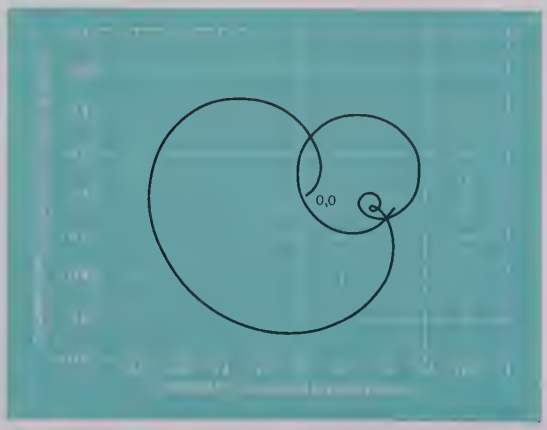

Figure 8 — Normalized transfer product of a four-pole network in terms of polar coordinates (Nyquist diagram).

Probably the most obvious way to record frequency information on a polar diagram is simply to mark known points of frequency on the polar display as the pen is traversing the locus of the rotating vector. One of the major disadvantages of this technique, in addition to the requirement for making frequency marks on the display, is the fact that, if the polar display goes to zero over some portion of the frequency range, it is extremely difficult to determine the exact range of frequencies over which this condition occurred. One technique of overcoming this difficulty is shown in Figure 10.

In this display, the *X* axis is driven proportional to the Co information plus the linear addition of a frequency component. The ordinate or *Y* axis is driven proportional to the Quad information plus the linear addition of the same frequency information. With this type of display, the origin of the Co/Quad graph, instead of remaining at one fixed point, now traverses a 45° line such that the center of the graph moves up and over as the presentation continues. The magnitude of the data vector is quickly determined by drawing a straight line from the data trace, at any annotated frequency, to the same frequency on the origin scale. Angular infor-

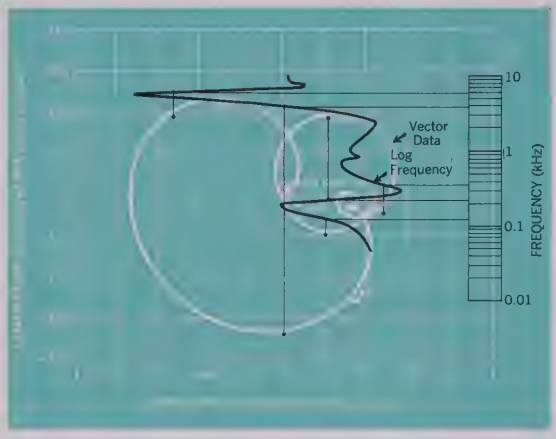

Figure 9 — A method for frequency identification of a polar display by using a two-pen X-Y recorder. The quadrature component, Y, and log frequency, Y', are recorded as a function of the coincident component, X.

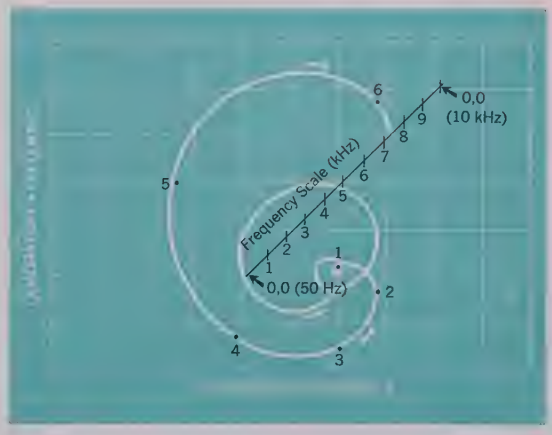

Figure 10 — A method for frequency identification of a polar display by adding a frequency constant to both the coincident and quadrature functions. The polar display origin travels on a 45° line from the initial origin at 0 Hz.

mation is derived as usual except that the position of the data on the vector graph is calculated with respect to the moving "frequency-sensitive" data origin. If, for any reason, the data should go to zero over a portion of the vector plot, this would be shown as a data line corresponding to the base frequency axis, and the frequencies at which the data again become non-zero are immediately apparent.

Work is also being done to present frequency information on a polar diagram as the product of the vector data and a dc voltage proportional to frequency. With this approach, the data origin of the display remains at a fixed point in the center of the graph. Discrete frequencies are then characterized by concentric circles expanding from the origin as frequency is increased.

Note that if either input A or B or both reduce in magnitude, the resulting polar display shrinks to a new radius. However, the phase angle which is being directly displayed will remain the same (for an essentially linear system).

Measurement of Mutual Properties Between Random Signals

When investigating a random signal for its information content, one of the most useful presentations is that of the signal's power spectrum or power spectral density.[4] Power spectrum estimates are useful in determining the effective energy seen by a specimen over a range of design frequencies, and also for indicating resonances that may have been excited because of random loading. When examining power spectra at two different points on a mechanical structure with the aim of trying to relate occurrences at the two different points, it is not sufficient to assume that peaks in the individual power spectra at or near the same frequency are necessarily the result of a signal transmission path or mutual property between the two points. For example, looking at Figures 11 and 12, we see two possible power spectra, each of which indicates significant energy in the area of 380 Hz. The natural reaction would be to relate the two points at which the spectra were measured as having a mutual path of transmission at least in the neighborhood of 400 Hz.

However, no definite statement can be made about the mutual properties of these two signals unless a coherent type of analysis between the signals is performed. Classically, this type of analysis would be either a cross-correlation analysis in the time domain or a cross-spectrum analysis in the frequency domain.

Cross-correlation and cross-power-spectrum measurements are related uniquely through a Fourier transform. For example, starting with the two signals $f_1(t)$ and $f_2(t)$, a cross-correlation function $\varphi_{12}(\tau)$ is defined as

$$\varphi_{12}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t) f_2(t+\tau)\, dt$$

The cross-spectral density function for the same two signals may then be stated as

$$P_{12}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \varphi_{12}(\tau) \epsilon^{-j\omega\tau}\, d\tau$$

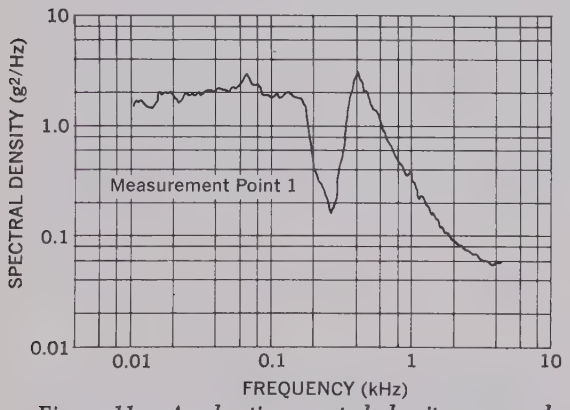

Figure 11 — Acceleration spectral density measured at point 1 on a structure under random excitation.

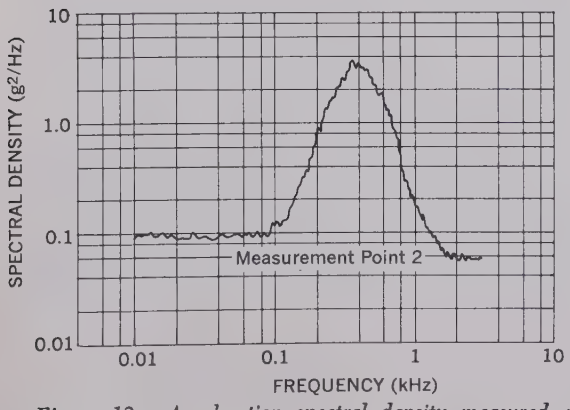

Figure 12 — Acceleration spectral density measured at point 2 on a structure under random excitation.

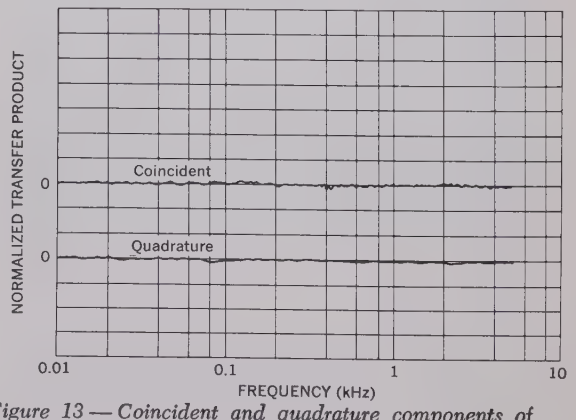

Figure 13 — Coincident and quadrature components of the cross-spectral density function between point 1 and 2 on a structure. The data show that no mutual relationships exist for the vibration spectra at the two points.

Since $P_{12}(\omega)$ is complex, it may be written either as

$$P_{12}(\omega) = C_{12}(\omega) - jQ_{12}(\omega)$$

where $C_{12}(\omega)$ is the Co component, and $Q_{12}(\omega)$ is the Quad component, or as

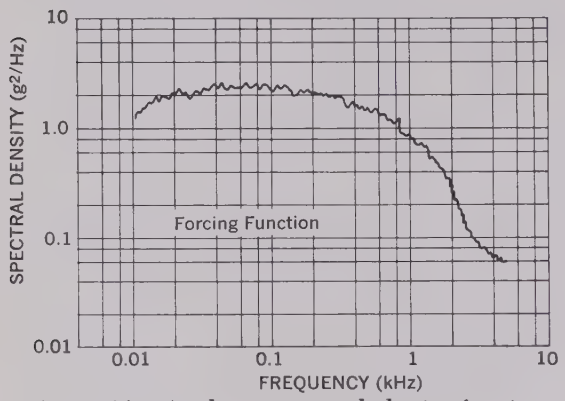

Figure 14 — Acceleration spectral density function of the random input to a structure.

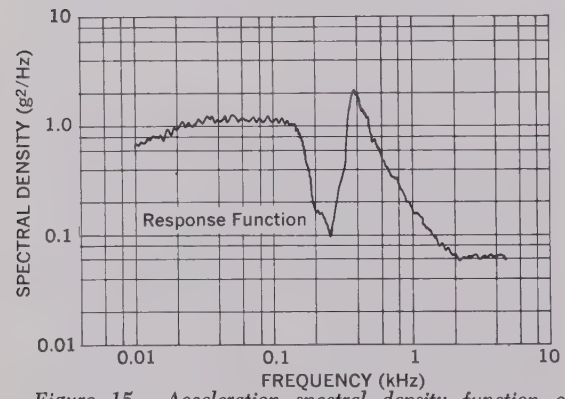

Figure 15 — Acceleration spectral density function of the response of a structure with three-degrees-of-freedom to the random excitation of Figure 14.

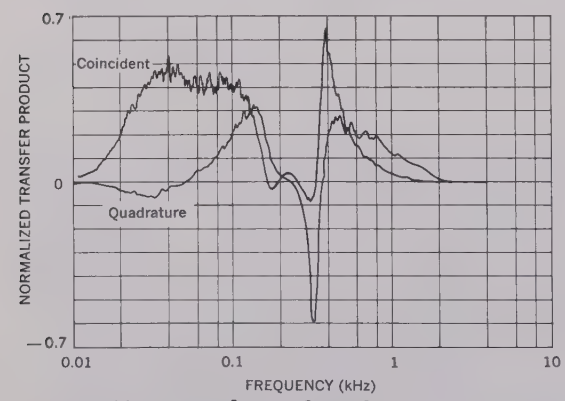

Figure 16 — Coincident and quadrature components of the cross-spectral density function between the input excitation and output response points on a structure having three-degrees-of-freedom. The data show that mutual relationships exist for the vibration spectra at the two measurement points.

$$P_{12}(\omega) = |P_{12}(\omega)| \angle \theta(\omega)$$

where

$$|P_{12}(\omega)| = [C_{12}(\omega)^2 + Q_{12}(\omega)^2]^{1/2}$$

and

$$\theta(\omega) = \tan^{-1} \frac{-Q_{12}(\omega)}{C_{12}(\omega)}$$

In other words, a cross-spectral density may be defined either as a magnitude function and a phase function or as Co and Quad components in the frequency domain. These are equivalent expressions. Figure 13 indicates the Co and Quad components of the cross-spectrum relationship between the two signals shown in Figures 11 and 12. Note that the mutual relationship which seemed apparent does not, in fact, exist.

Figures 14 and 15 indicate two power spectral density estimates which, on the surface, show no strong resemblance. However, the fact that they do indeed possess a mutual relationship is shown in Figure 16, where again the cross spectrum between signals in terms of Co and Quad components is displayed. Here we see that a very definite relationship exists between the two points at which random signal analyses have been performed.

An additional example of the relationships between random signals which may be revealed by using cross-spectrum analysis techniques is shown by comparing Figures 16 and 17. In Figure 16, the cross-spectral density function in terms of Co and Quad components is shown for a three-degree-of-freedom structure excited by a random input. Note that the relationship shown in Figure 16 is identical to the Co/Quad response shown in Figure 17 when the same structure was excited sinusoidally. This must be so, since, for linear systems, the actual transfer function of the system is independent of the forcing function. In other words, it does not change as the forcing input characteristics are changed.

For a system with a transfer function $H(f)$, the

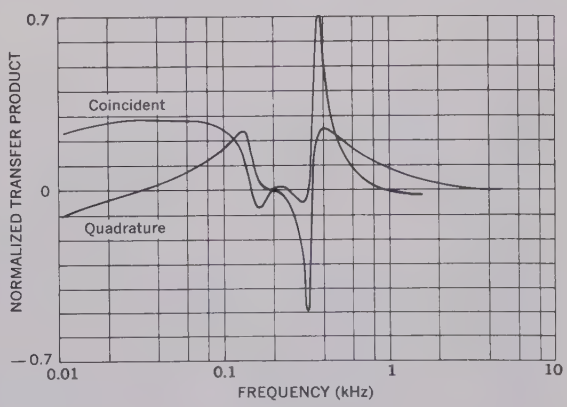

Figure 17 — Coincident and quadrature components of the transfer ratio function between the input and output of a structure, having three-degrees-of-freedom, with sinusoidal excitation.

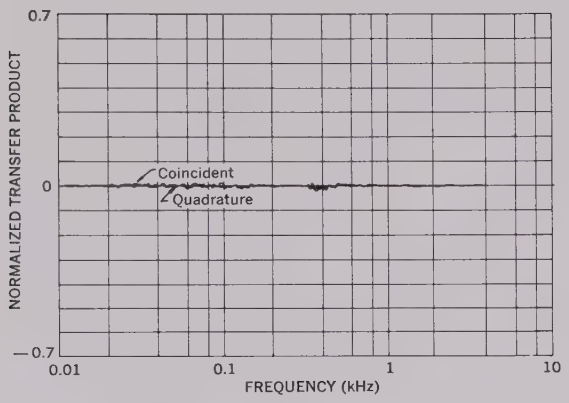

Figure 18 — Coincident and quadrature components of the cross-spectral density function between two independent and incoherent random forcing functions. The data show that no mutual properties exist between the two functions.

relationships between an input power spectrum, $P_1(f)$, an output power spectrum, $P_2(f)$, and a cross spectrum $P_{12}(f)$ are as follows:

$$P_2(f) = |H(f)|^2 P_1(f)$$

and

$$P_{12}(f) = P_1(f) H(f)$$

This says that the magnitude of the transfer function $|H(f)|$ can be obtained by knowing only the input and output power spectra, but that in order to obtain the complete transfer function including phase information, a cross-spectrum analysis must be performed in addition to knowing the input or output power spectrum.

Figure 18 indicates the transfer of energy between the random structural excitation of Figure 14 and an independent random forcing function. Obviously, since signals from independent and incoherent random sources are being compared, the mutual properties are zero, as is shown by the zero cross-spectrum properties of Figure 18.

Energy Measurements

As a further example, the transfer characteristics of a single-pole, RC, low-pass filter will be examined. The transfer function, in terms of magnitude and phase for a single-pole filter, is shown in Figure 19. The amplitude transfer characteristic begins at unity gain near dc and asymptotically approaches a 6-dB-per-octave rolloff. The phase characteristics begin in phase and approach a condition of 90° or simply $j\omega$. The same information in terms of Co and Quad products is shown on Figure 20. Here the Co function begins at a maximum, indicating true in-phase conditions, and the Quad function peaks up at the point where Co and Quad are equal. This point, representing 45°, or an arc tangent of 1, is also the 3-dB point of the filter pass band. Figure 21 indicates the Co/Quad relationship as shown in a

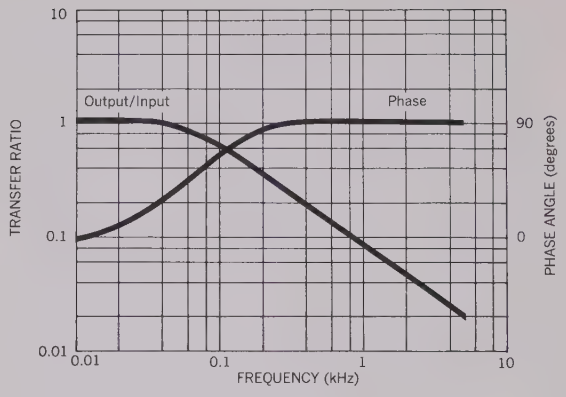

Figure 19 — Transfer function of a single-pole filter in terms of magnitude ratio and phase angle.

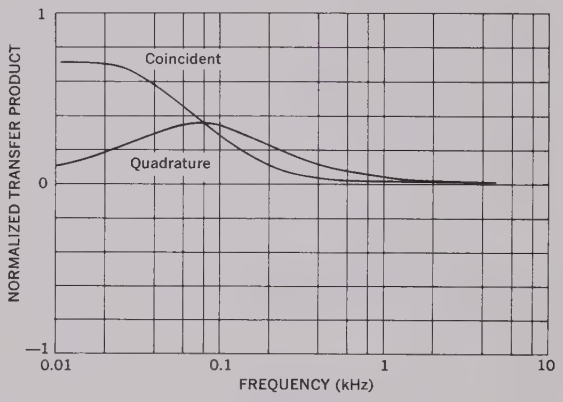

Figure 20 — Normalized transfer product of a single-pole filter in terms of the coincident and quadrature components of the transfer ratio vector.

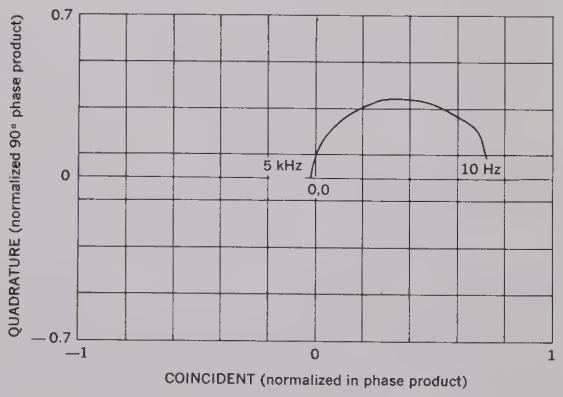

Figure 21 — Normalized transfer product of a single-pole filter in terms of polar coordinates.

polar diagram. Since the phase relationship never exceeds 90° between the input and output of this filter, the polar representation is confined to the first quadrant.

Another application for real and imaginary component analysis exists in energy measurements. For example, to calculate the effectiveness of different

absorbing materials, the following procedure can be used: If a force acting over a known distance describes the amount of work done and the rate of change of work with time is energy, the force operating with a known velocity then describes an energy term. That is, if a known force is applied to an item and the resultant velocity is measured at the point of application, information relating to the total energy per second put into the specimen is available.

The real component of this energy term then represents energy absorbed by the damping material, and the imaginary or quadrature term represents energy stored in the damping material. This is shown in Figure 22.

By making repeated Co/Quad analyses for different materials, an optimum fit can be achieved between resonant properties of the material under investigation and a minimum imaginary component. If the imaginary component can be reduced to zero, perfect isolation or complete damping has been achieved.

Analysis Systems

A means of automatically calculating the real and imaginary (product) relationships between two analog signals is available by using the SD109A Co/Quad Analyzer shown in Figure 23. The SD109A is used in conjunction with two channels of Spectral Dynamics Corporation Dynamic Analyzers as shown in Figure 24. The analyzers are tuned by a reference sinusoid either directly during system excitation (on line), or when playing back taped or off-line data at a later time (swept spectrum mode). Only a narrow band of signals, centered in frequency around the reference sinusoid, is passed by the analyzer. The narrow-band output retains all the phase and amplitude intelligence of the original input at the tuning frequency.

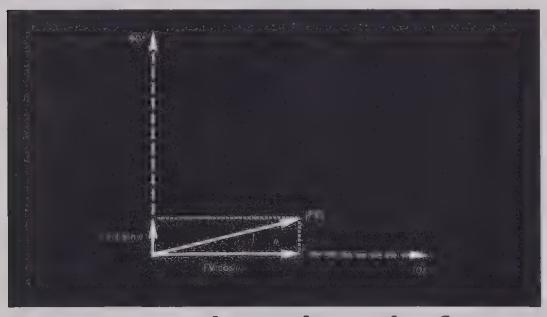

Figure 22 — Vector diagram showing the effectiveness of energy absorbing materials. The real component of the vector represents energy absorbed by the damping material. The imaginary component represents energy stored in the material.

An additional output from the analyzer is provided at the analyzer intermediate frequency (IF) of 100 kHz. The IF output is always centered about 100 kHz, regardless of the audio frequency to which the analyzer is tuned. The IF output is also phase- and amplitude-coherent with the fundamental of the signal input.

Two such filtered IF waveforms are used as signal inputs to the SD109A. Within the SD109A, these signals are buffered and heterodyned to 1 kHz, as shown in Figure 25. One of the 1-kHz signals is then phase-shifted 90 degrees, producing three simultaneous 1-kHz filtered waveforms. (Note that, regardless of the audio frequency of the signals under analysis, the three signals described above are always fixed in frequency at 1 kHz.)

Simultaneous multiplication of signals with and without phase shift is then performed, producing the trigonometric expansions shown in Figure 25. Adjustable time averaging is provided in each channel (especially useful when examining narrow-band random signals), and the bipolar dc terms $AB \cos \theta$ and $AB \sin \theta$ are presented both as front panel meter indications and as signal outputs suitable for recording.

The use of fixed-frequency IF signals for analysis not only optimizes phase shift and multiplier requirements to one frequency, but also provides an extremely wide frequency range of operation from 1 Hz to well in excess of 30 kHz.

Figure 23 — Spectral Dynamics Corp. Model SD109A Co/Quad Analyzer.

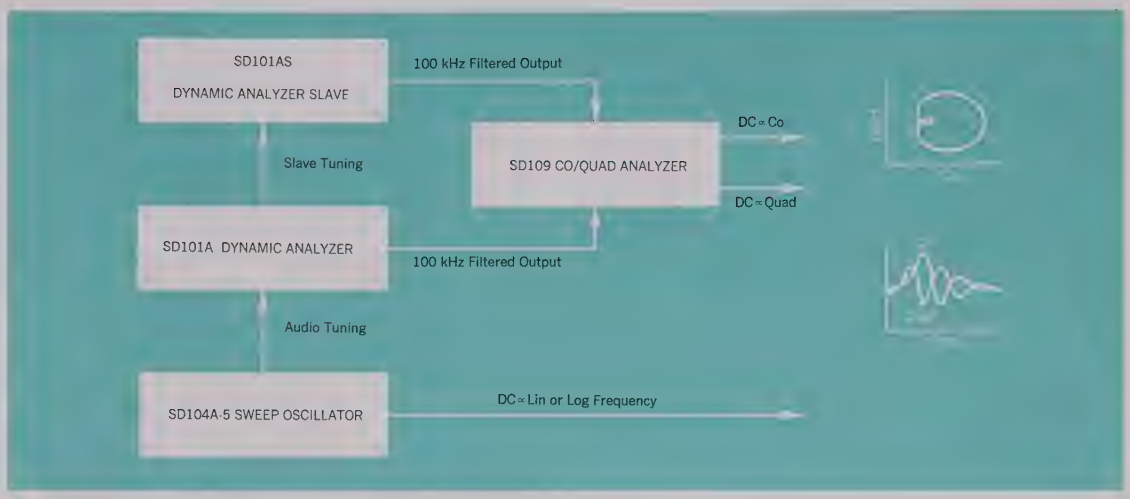

Figure 24 — Typical system configuration for modal and cross-spectrum analysis techniques.

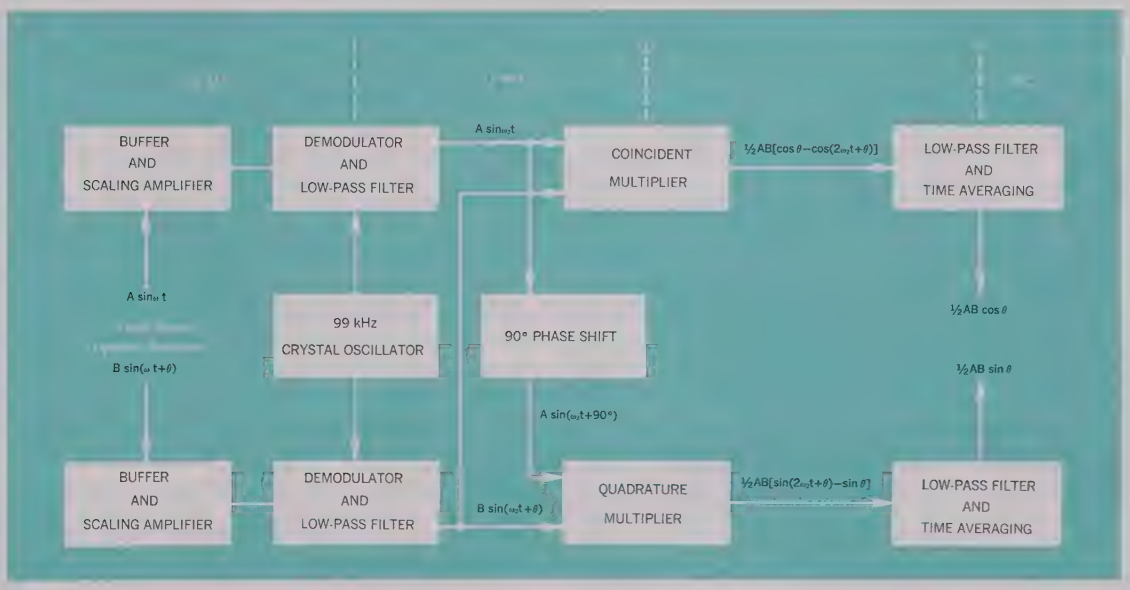

Figure 25 — Block diagram of the SD109A Co/Quad Analyzer.

Summary

In summary, it is seen that the definition of mutual properties between signals in terms of real and imaginary components has significant advantages in three specific areas. These are:

☐ The definition of modal properties of a large structure

☐ The unambiguous definition of the phase between two signals as a function of frequency

☐ The mutual relationships between random signals in the frequency domain

Coincident and quadrature components also find significant applications in energy and mechanical impedance measurements (although in the latter, either Co or Quad can normally be used in favor of a direct phase presentation). In general, any time a transfer measurement through a complex system is required, serious consideration should be given to the use of real and imaginary component vectors.

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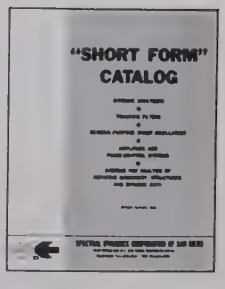

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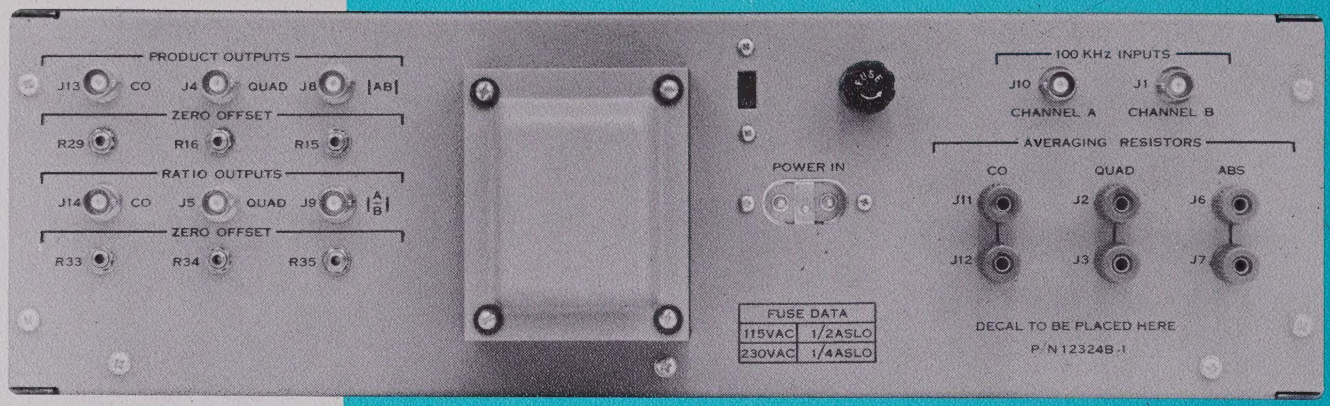

REAR VIEW MODEL SD109B

SPECIFICATIONS

SYSTEM ANALYSIS RANGE

2 Hz to 50 kHz (Using SDC Dynamic Analyzer or Tracking Filter units)

INPUTS (EACH OF TWO CHANNELS)

Frequency ..... 100 kHz (fixed)
Amplitude ..... 31.6 mV to 3.16 V rms for full scale output. 40 dB input attenuator calibrated in 5 dB steps
Impedance ..... 10 Kilohms

OUTPUTS

Product Mode .... −10 V to +10 Vdc for both CO and QUAD functions
0 to +10 Vdc for absolute product
Dynamic Range 60 dB
Accuracy ..... ±0.25% of full scale ±2.5% of reading
Ratio Mode ..... −100 mV to +100 mV dc for both CO and QUAD functions; 0 to 100 mV for absolute ratio
Dynamic Range 50 dB
Accuracy ..... ±2% of full scale ±5% of reading
Ratio Ranges ... Ratio 0.1 to 100 in three ranges
Ratio 1 . . . B minimum ≧0.1B full scale
Ratio 10,100 . . . B minimum ≧0.01B full scale
A input may vary 0 to −50 dB.
Square Mode .... 0 to +10 Vdc for both CO and QUAD functions
Dynamic Range 80 dB
Accuracy ..... ±0.1% of full scale ±2.5% of reading
Impedance ..... Less than 600 ohms

MISCELLANEOUS

Meters ..... Front panel meters continuously monitor CO and QUAD components. Meters are graduated in 100 divisions between limits of +10 and −10
Linearity .... 2% of full scale
Time Averaging . Continuously adjustable 0.1 to 10 seconds, ±15% for each channel. Easily expandable to 100 seconds thru addition of external resistance.
Connectors ..... BNC Connectors on rear panel
Power ..... 105-125/210-250 Vac, 50-60 Hz, at ≈60W; input line voltage switch selectable at rear of chassis
Dimensions ..... *Height* ..... 5¼" (13.3 cm)
Width ..... 19" (48.3 cm)
Depth ..... 14" (35.6 cm)
Shipping Weight ..... ≈20 lbs (9 kg)

ORDERING INFORMATION

The SD109B is designed as an add-on building block for use with the SD101A, SD101B, SD1012 or SD1012B tracking filters. For optimum operation, the tracking filters used with the SD109B should employ matched pairs of plug-in filter elements of the same bandwidths. Matched pairs of filters are available from Spectral Dynamics.

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Specifications subject to change without notice

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